ANALYSIS OF STEADY THREE–DIMENSIONAL HYDROMAGNETIC STAGNATION POINT FLOW TOWARDS A STRETCHING SHEET WITH HEAT GENERATION

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Abstract. This paper is concerned with the three-dimensional hydromagnetic stagnation point flow towards a stretching sheet with heat generation. It is assumed that a uniform magnetic field is applied normal to the plate which is maintained at a constant temperature. The coupled partial differential equations are reduced into ordinary differential equations by using similarity transformations. The series solutions of the coupled non-linear system is obtained using an analytical technique namely the homotopy analysis method (HAM). We use a two-stage method, where both the convergence

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control parameter and the initial guess are optimally selected to minimize the residual error due to the approximation. To do the latter, we consider a family of initial guesses parameterized by a constant which gives the decay rate of the solutions. In several cases considered, we are able to obtain solutions with extremely small residual errors after relatively few iterations are computed. The convergence, salient features of the flow and heat transfer characteristics are analyzed and discussed in detail through graphs. We also discuss the effect of the strength of the uniform magnetic field, the surface stretching velocity, and the heat generation/absorption coefficient on both the flow and heat transfer.

Keywords: boundary layer; heat generater; homotopy analysis method; homotopy solution; laminar flow; stretching sheet.

1. Introduction

Stagnation-point flow is a topic of significance in fluid mechanics, in the sense that stagnation points appear in virtually all flow fields of science and engineering. In some situations flow is stagnated by a solid wall, while in others a free stagnation point or a line exists in the interior of the fluid domain. After Weidman and Putkaradze [1], these flows may be characterized as inviscid or viscous, steady or unsteady, two-dimensional or three-dimensional, symmetric or asymmetric, normal or oblique, homogeneous or two fluid, and forward or reverse. The flow near a stagnation point has an important bearing on several technological processes. Such processes are cooling of nuclear reactors during emergency shutdown, heat exchangers placed in a law-velocity environment, cooling of electronic devices by fans, solar central receivers exposed to wind currents and many hydrodynamic processes [2].

The two-dimensional flow of a fluid near a stagnation point is a classical problem in fluid dynamics and its solution was first given by Hiemenz [3]. Later, the problem of stagnation point flow was extended numerically by Schlichting and Bussmann [4] and analytically by Ariel [5] to include the effect of suction. The problem of stagnation point flow was extended in numerous ways to include various physical effects. The axisymmetric three dimensional stagnation point flow was first studied by Homann [6]. Howarth [7] and Davey [8] considered the three-dimensional orthogonal stagnation-point flow of which the axisymmetric case was studied as a special case. Yeckel et al. [9] showed that the classical Homann flow can be used to supply boundary conditions for Reynolds' lubrication equations for flow within a thin viscous film, thereby leading to a model for the thinning of the film in the stagnation region of a turbulent water jet. Wang [10] studied the stagnation-point flow with surface slip that occurs in rarefied flow or under high pressure, while Blyth and Pozrikidis [11] investigated the stagnation-point flow of a Newtonian fluid against a Newtonian liquid film resting on a plane wall. They individually considered several cases for orthogonal two-dimensional, axisymmetric, three-dimensional and oblique two-dimensional flow.

In the present study the steady and hydromagnetic laminar axisymmetric three dimensional stagnation-point flow of an incompressible viscous fluid impinging on a permeable stretching surface will be considered. Such flows of gases or liquids impinging on a permeable surface have several industrial applications which encompass the need to enhance heat and/or mass transfer. Typical examples are associated with the annealing of metals, cooling of gas turbine blades, cooling in grinding processes and cooling of photovoltaic cells [12]. The shear stress induced by the impinging fluid stream is utilized in surface cleaning, paper and photographic film manufacturing, wire coating and finishing of metal strips [13].

The steady hydromagnetic laminar axisymmetric three dimensional stagnation point flow of an incompressible viscous fluid impinging on a permeable stretching surface with heat generation/absorption was studied by Attia [14]. He applied a uniform magnetic field directed normal to the plate where the induced magnetic field was neglected. The wall and stream temperatures were assumed to be constants. This model come as a coupled system of nonlinear differential equations and was numerically solved.

Liao [15] used the concept of homotopy to propose an analytic method for highly nonlinear problems, namely the homotopy analysis method (HAM) (see [16], [17]). Different from perturbation techniques, HAM does not require small parameters, so that it is valid for highly nonlinear problems, especially for those with small or large physical parameters (see [18]). Different from all previous analytic methods, HAM always gives us a family of series solutions whose convergence region can be adjusted and controlled by an auxiliary parameter $\hbar$. This parameter can be obtained in different ways such as an $\hbar$–curve plot or by minimizing the residual error [19], [20], [21], [22], [23], [24]. HAM has been successfully applied to many types of nonlinear problems in science and engineering [25], [26], [27], [28], [29], [30], [31], [32], [33], [34, 35].

The aim of the present work is to study analytically the governing momentum and energy equations of the steady three-dimensional hydromagnetic stagnation point flow towards a stretching sheet with heat generation derived in [14]. Homotopy analysis method (HAM) is investigated in obtaining the analytical solution. In order to obtain accurate approximate analytical solutions in the whole spatial region, we consider multiple initial guesses, in order to find the best initial guess which permits convergence and accuracy after relatively few terms are calculated. We also select the convergence control parameter optimally, through the construction of an optimal control problem for the minimization of the accumulated residual errors. To the best of our knowledge, this is the first work that provides an analytic solution for this problem.

2. Formulation of the problem

The steady three-dimensional stagnation point flow of a viscous incompressible fluid near a stagnation point at a surface coinciding with the plane $z = 0$ is considered where the flow is in the region $z > 0$. Two equal and opposing forces are applied along the radial direction so that the surface is stretched, keeping the origin fixed. Cylindrical coordinates r, ϕ, z are used with assumptions that the wall is at $z = 0$, the stagnation point is at the origin and that the flow is in the direction of the negative z-axis. We denote the radial and axial velocity components in frictionless flow by U and W, respectively, whereas those in viscous flow will be denoted by $u = u(r; z)$ and $w = w(r; z)$ where the component in the ϕ direction vanishes. A uniform magnetic field B_0 is applied normal to the plate where the induced magnetic field is neglected by assuming very small magnetic Reynolds number. For three-dimensional flow let the fluid far from the plate, as z tends infinity, be given by the potential flow

$$U = ar, \qquad W = -2az, \tag{2.1}$$

where $a > 0$ is a constant characterizing the velocity of the mainstream flow. Then, from Euler equation the pressure distribution will be

$$p = p_0 - \frac{\rho a^2}{2}(r^2 + 4z^2),$$

where ρ is the density of the fluid and p_0 is the pressure at the stagnation point. Thus the continuity and momentum equations for the three dimensional steady state flows, using the usual boundary layer approximations reduce to

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{2.2}$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial \rho}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) + \sigma B_0^2(U(r) - u), \tag{2.3}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial \rho}{\partial r} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 w}{\partial z^2}\right), \tag{2.4}$$

where μ is the coefficient of viscosity of the fluid and σ is the electrical conductivity of the fluid. With boundary conditions

$$\begin{aligned} z = 0 : u &= cr, w = 0, \\ z \to \infty : \quad & u \to ar, \end{aligned} \tag{2.5}$$

where c is a positive constant related to the stretching velocity.

The governing boundary layer equation of energy, neglecting the dissipation, with temperature dependent heat generation or absorption is

$$\rho c_\rho\left(u\frac{\partial \theta}{\partial r} + w\frac{\partial \theta}{\partial z}\right) = k\frac{\partial^2 T}{\partial z^2} + Q(T - T_\infty), \tag{2.6}$$

subject to the boundary conditions

$$\begin{aligned} z = 0 : T &= T_w, \\ z \to \infty : T &\to T_\infty. \end{aligned} \tag{2.7}$$

Using the similarity solution

$$u = crf'(\eta), \quad w = -2\sqrt{cv}f(\eta), \quad \eta = \sqrt{c/vz}, \tag{2.8}$$

where $v = \mu/\rho$ is the kinematic viscosity of the fluid and prime denotes differentiation with respect to η, equations (2.2)-(2.4) subject to the boundary conditions (2.5) can be

$$f''' = -2ff'' + f'^2 - C^2 - Ha^2(C - f') \tag{2.9}$$
$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = C, \tag{2.10}$$

where $Ha^2 = \sigma B_0^2/c\rho$, Ha is the modified Hartmann number and $C = a/c$ is the stretching parameter. Using equation (2.8) with the help of the non-dimensional variable

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$

we can reduce equations (2.6) and (2.7) to

$$\theta'' + 2Prf\theta' + PrB\theta = 0 \tag{2.11}$$
$$\theta(0) = 1, \quad \theta(\infty) = 0, \tag{2.12}$$

where $Pr = \mu c_\rho/k$ is the prandtl number and $B = Q/c\rho c_\rho$ is the dimensionless heat generation/absorption coefficient.

3. Mathematical analysis for HAM solution

Based on the boundary conditions (2.10) and (2.12), the solution can be expressed by a set of base functions

$$\{\eta^k e^{-s\eta} | k, s \geq 0\}$$

in the form

$$\begin{aligned} f(\eta) &= a_{1,0} + \sum_{k=0}^{+\infty}\sum_{s=0}^{+\infty} a_{1,k}\eta^k e^{-s\eta}, \\ \theta(\eta) &= a_{2,0} + \sum_{k=0}^{+\infty}\sum_{s=0}^{+\infty} a_{2,k}\eta^k e^{-s\eta}, \end{aligned} \tag{3.1}$$

where $a_{1,k}$ and $a_{2,k}$ ($k = 1, 2, \ldots$) are coefficients to be determined. Moreover, according to the *Rule of Solution Expression* denoted by (3.1) and the boundary conditions (2.10) and (2.12), it is natural to choose initial approximations of $f(\eta)$ and $\theta(\eta)$ in the form

$$f_0(\eta) = (1 - C)/\gamma + C\eta + \frac{(C - 1)e^{-\gamma\eta}}{\gamma}, \quad \theta_0(\eta) = e^{-\beta\eta},$$

where γ and β are constants to be determined later. Based on the governing equations (2.9) and (2.11) and the *Rule of Solution Expression* (3.1), it is reasonable to choose the linear operators

$$L_f[\hat{f}(\eta;q)] = \hat{f}'''(\eta;q) + \gamma \hat{f}''(\eta;q), \tag{3.2}$$

$$L_\theta[\hat{\theta}(\eta;q)] = \hat{\theta}''(\eta;q) + \beta\hat{\theta}'(\eta;q), \tag{3.3}$$

with the properties

$$L_f[C_1 + C_2\eta + \frac{C_3e^{-\gamma\eta}}{\gamma^2}] = 0, \quad L_\theta[C_4 - \frac{C_5e^{-\beta\eta}}{\beta}] = 0, \tag{3.4}$$

where C_i $(i = 1, 2, \ldots, 5)$ are the integration constants that will be determined by the boundary conditions. From the equations (2.9) and (2.11), we define the nonlinear operators N_f and N_θ as

$$N_f[\hat{f}(\eta;q)] = \hat{f}'''(\eta;q) - \hat{f}'^2(\eta;q) + 2\hat{f}(\eta;q)\hat{f}''(\eta;q) + C^2 + Ha^2(C - \hat{f}'(\eta;q)),$$
$$N_\theta[\hat{f}(\eta;q), \hat{\theta}(\eta;q)] = \hat{\theta}''(\eta;q) + 2Pr\hat{f}(\eta;q)\hat{\theta}'(\eta;q) + PrB\hat{\theta}(\eta;q)).$$

According to the underlying principle of the HAM [36, 37], the *zeroth-order deformation* problems can be constructed as follows:

$$(1-q)L_f[\hat{f}(\eta;q) - f_0(\eta)] = q\hbar N_f[\hat{f}(\eta;q)], \tag{3.5}$$

$$(1-q)L_\theta[\hat{\theta}(\eta;q) - \theta_0(\eta)] = q\hbar N_\theta[\hat{\theta}(\eta;q), \hat{f}(\eta;q)], \tag{3.6}$$

subject to the boundary conditions

$$\begin{aligned} &\hat{f}(0;q) = 0, \quad \hat{f}'(0;q) = 1, \quad \hat{\theta}(0;q) = 1, \\ &\hat{f}'(\eta;q) = C, \quad \hat{\theta}(\eta;q) = 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{3.7}$$

In the *zeroth-order deformation equations* (3.5)-(3.6), $q \in [0, 1]$ is an embedding parameter and $\hbar$ is the auxiliary parameter. For $q = 0$ and $q = 1$, the above *zeroth-order deformation* equations (3.5)-(3.6) have the solutions

$$\hat{f}(\eta;0) = f_0(\eta), \quad \hat{\theta}(\eta;0) = \theta_0(\eta), \tag{3.8}$$

$$\hat{f}(\eta;1) = f(\eta), \quad \hat{\theta}(\eta;1) = \theta(\eta). \tag{3.9}$$

When q increases from 0 to 1, then $\hat{f}(\eta;q)$ and $\hat{\theta}(\eta;q)$ vary from the initial guesses $f_0(\eta)$ and $\theta_0(\eta)$ to the exact solutions $f(\eta)$ and $\theta(\eta)$, respectively.

Setting

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m \hat{f}(\eta;q)}{\partial q^m}\bigg|_{q=0},$$

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \hat{\theta}(\eta;q)}{\partial q^m}\bigg|_{q=0},$$

and expanding $\hat{f}$ and $\hat{\theta}$ into the Taylor series expansion with respect to the embedding parameter q, we have

$$\hat{f}(\eta;q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q^m, \tag{3.10}$$

$$\hat{\theta}(\eta;q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) q^m. \tag{3.11}$$

If the auxiliary parameter $\hbar$ is chosen in such a way that these series are convergent at $q = 1$, we arrive at

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{3.12}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta). \tag{3.13}$$

In this line we define the mth-order derivative of ϕ as

$$D_m(\phi) = \frac{1}{m!} \frac{\partial^m(\phi)}{\partial q^m}\bigg|_{q=0}.$$

Using the properties of the mth-order derivative [36], we obtain

$$\begin{aligned}
D_m\{(1-q)L_f[\hat{f}(\eta;q) - f_0(\eta)]\} &= D_m\{L_f[\hat{f}(\eta;q) - q\hat{f}(\eta;q) + f_0(\eta)q - f_0(\eta)]\} \\
&= L_f[D_m(\hat{f}(\eta;q)) - D_m(q\hat{f}(\eta;q)) + f_0(\eta)D_m(q)] \\
&= L_f[f_m(\eta) - f_{m-1}(\eta) + f_0(\eta)D_m(q)] \\
&= \begin{cases} L_f[f_m], & m \le 1 \\ L_f[f_m - f_{m-1}], & m > 1 \end{cases} \\
&= L_f[f_m - \chi_m f_{m-1}], \qquad (3.14)
\end{aligned}$$

where

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases}$$

Apply the operator D_m to the *zeroth-order deformation equation* (3.5), to have

$$D_m\{(1-q)L_f[\hat{f}(\eta) - f_0(\eta)]\} = \hbar D_{m-1}\{N_f[\hat{f}(t;q)]\},$$

where

$$\begin{aligned}
D_{m-1}\{N_f[\hat{f}(t;q)]\} &= f'''_{m-1}(\eta) + 2\sum_{i=0}^{m-1} f_i(\eta) f''_{m-1-i}(\eta) \\
&\quad - \sum_{i=0}^{m-1} f'_i(\eta) f'_{m-1-i}(\eta) - Ha^2 C f'_{m-1}(\eta) + (C^2 + Ha^2 C)(1-\chi_m).
\end{aligned}$$

Then the *mth-order deformation equation* related to f_m becomes

$$\begin{aligned} L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] &= \hbar \Bigg(f'''_{m-1}(\eta) + 2\sum_{i=0}^{m-1} f_i(\eta) f''_{m-1-i}(\eta) - \sum_{i=0}^{m-1} f'_i(\eta) f'_{m-1-i}(\eta) \\ &\quad - Ha^2 C f'_{m-1}(\eta) + (C^2 + Ha^2 C)(1-\chi_m)\Bigg), \end{aligned}$$

or what amounts to the same as

$$\begin{aligned} f_m &= \hbar L_f^{-1}\Bigg[f'''_{m-1}(\eta) + 2\sum_{i=0}^{m-1} f_i(\eta) f''_{m-1-i}(\eta) - \sum_{i=0}^{m-1} f'_i(\eta) f'_{m-1-i}(\eta) \\ &\quad - Ha^2 C f'_{m-1}(\eta) + (C^2 + Ha^2 C)(1-\chi_m)\Bigg] + \chi_m f_{m-1}. \end{aligned} \tag{3.15}$$

Applying similar procedure for equation (3.6), we obtain the *mth-order deformation equation* related to θ_m as

$$\theta_m = \hbar L_\theta^{-1}\left[\theta''_{m-1} + 2Pr\sum_{i=0}^{m-1} f_i \theta'_{m-1-i} + PrBf_{m-1}\right] + \chi_m \theta_{m-1}, \tag{3.16}$$

with boundary conditions

$$\begin{aligned} &f_m(0) = 0, \quad f'_m(0) = 0, \quad \theta_m(0) = 0 \\ &f'_m(\eta) = 0, \quad \theta_m(\eta) = 0 \qquad \text{as } \eta \to \infty. \end{aligned} \tag{3.17}$$

The general solutions of equations (3.15)-(3.16) are

$$\begin{aligned} f_m(\eta) &= f_m^*(\eta) + C_1 + C_2\eta + \frac{C_3 e^{-\gamma\eta}}{\gamma^2}, \\ \theta_m(\eta) &= \theta_m^*(\eta) + C_4 - \frac{C_5 e^{-\beta\eta}}{\beta}, \end{aligned}$$

where $f_m^*(\eta)$ and $\theta_m^*(\eta)$ are the particular solutions and the constants C_i $(i = 1, 2, 3, 4)$ are determined by the boundary conditions (3.17).

This way, it is easy to solve the linear non-homogeneous equations (3.15) and (3.16) with boundary conditions (3.17) by using symbolic computational software such as Mathematica. Thus, the mth-order approximate analytic solutions for the system of equations (2.9)-(2.10) and (2.11)-(2.12) are

$$f(\eta) \simeq \sum_{i=0}^{m} f_i(\eta), \tag{3.18}$$

$$\theta(\eta) \simeq \sum_{i=0}^{m} \theta_i(\eta). \tag{3.19}$$

4. Results and discussion

The series given by equations (3.18) and (3.19) are the analytical solutions of the present problem of laminar fluid flow on a permeable stretching sheet. The convergence region and rate of approximation of these series solutions depend upon the auxiliary parameter $\hbar$. Firstly, we consider a family of initial guesses parameterized by a constant which gives the decay rate of the solutions. In several cases considered, we are able to obtain solutions with extremely small residual errors after relatively few iterations are computed. In order to find the admissible values of this parameter, the $\hbar$-curves of $f''(0)$ and $\theta'(0)$ are displayed in Fig.1 when $\gamma = 7$ and $\beta = 5$. These $\hbar$-curves have been drawn for the physical quantities such as the reduced skin friction coefficient $f''(0)$ and the wall heat transfer rate $-\theta'(0)$.

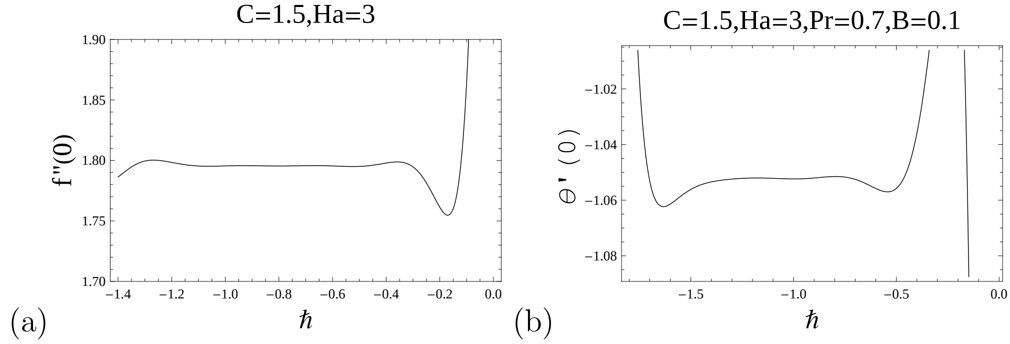

Figure 1: The $\hbar$-curve for (a) Eq. (3.18), (b) Eq. (3.19)

In our approach $\hbar$ is an optimal convergence-control parameter that can be used to accelerate the convergence of the homotopy-series solution. At the m-th order of approximation, one can define the square residual error

$$\Delta_m = \int_0^{+\infty} \left(N \left[\sum_{i=0}^{m} f_i(\eta) \right] \right)^2 d\eta. \tag{4.1}$$

Note that Δ_m contains $\hbar$ as an unknown parameter. At a given order of approximation m, the optimal value of $\hbar$ is given by the minimum of Δ_m, corresponding to the nonlinear algebraic equation

$$\frac{d\Delta_m}{d\hbar} = 0.$$

The optimal value of $\hbar$ when $m = 10$ is -0.97858.

The residual error of the solution is given via Fig. 2. The velocity profile of f and f' are respectively presented in Figs. 3 and 4.

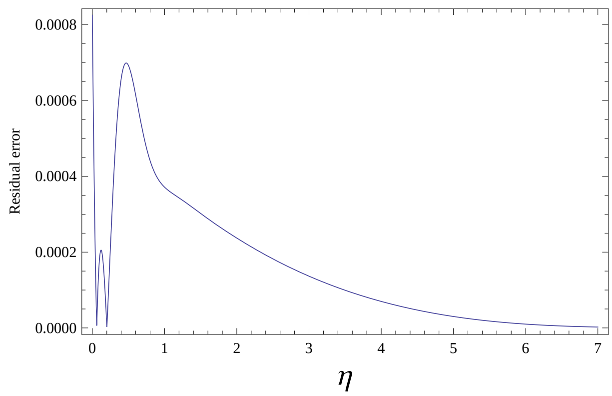

Figure 2: The residual error of the solution using 10-term

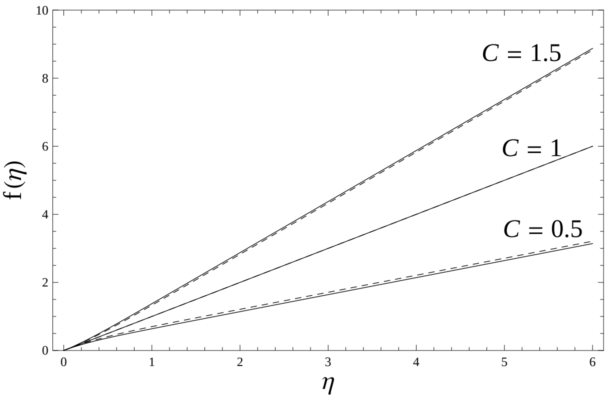

Figure 3: The profile of $f(\eta)$ with different values of Ha and C. Dash line for $Ha = 0$, solid line for $Ha = 3$

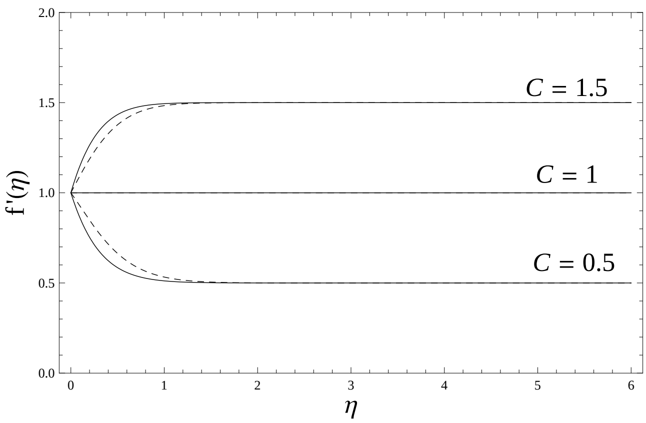

Figure 4: The velocity profile for $f'(\eta)$ with different values of Ha and C. Dash line for $Ha = 0$, solid line for $Ha = 3$

The figures show that as the modified Hartmann number (Ha) or the stretching parameter (C) increase then the velocity profile of f and f' are increased. The effect of Ha on both f and f' depends on C. For $C < 1$, increasing Ha decreases f and f' while for $C > 1$, increasing Ha increases them. The figures indicate also that the effect of C on f and f' is more pronounced for smaller values of Ha. Also, increasing C decreases the velocity boundary layer thickness. The profile of temperature θ for several values of Pr and B is presented in Fig. 5 with $Pr = 0.7$ and $B = 0.1$.

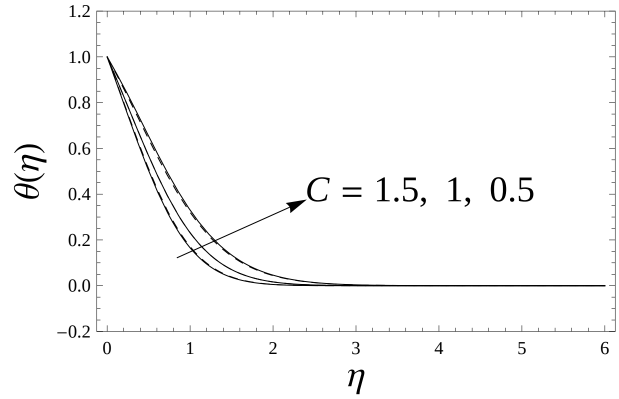

Figure 5: Temperature profile for several values of Ha and C with $Pr = 0.7$, $B = 0.1$. Dash line for $Ha = 0$, solid line for $Ha = 3$

It is clear that while C increasing θ decreasing, and for small value of Ha the effect of C becomes more apparent. The thermal boundary layer thickness decreases as C increases. Moreover, small C give more effect for the decreasing of θ as Ha increasing.

The efficacy of the Prandtl number (Pr) and C are studied in Fig. 6.

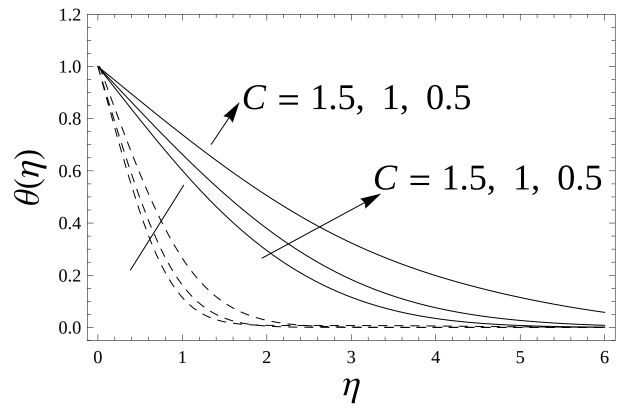

Figure 6: Temperature profile for several values of Pr and C with $Ha = 1$, $B = 0.1$. Dash line for $Pr = 1$, solid line for $Pr = 0.1$

It is clear that the θ increases as C decreases, also increasing Pr makes θ and thermal boundary layer thickness decrease.

Finally, Fig. 7 shows the effect of C and dimensionless heat generation coefficient (B) on the temperature profile for $Ha = 0.5$ and $Pr = 0.7$. When B increases, then the temperature θ and the boundary layer thickness increase. The effect of B on θ is more pronounced for smaller C. However, the effect of C on θ is more apparent for higher B.

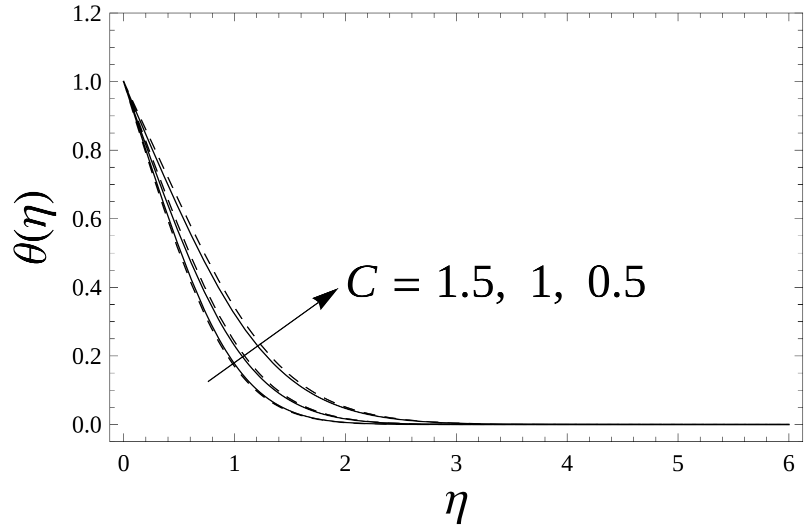

Figure 7: Temperature profile for several values of B and C with $Ha = 0.5$, $Pr = 0.7$. Dash line for $B = 0.1$, solid line for $B = -0.1$

5. Conclusions

The three dimensional hydromagnetic stagnation point flow of a viscous incompressible fluid impinging on a permeable stretching surface is studied in the presence of uniform magnetic field with heat generation/absorption. By a powerful and newly developed technique, based on the homotopy analysis method, the convergence series solutions are obtained. The proper values for $\hbar$, the proper initial guess and the proper linear operator were chosen to give more accuracy of the homotopy solution. Graphs are plotted to analyze the variation of the pertinent flow parameters including the modified Hartmann number Ha, the stretching velocity, the heat generation/absorption parameter and the Prandtl number Pr. From the present analysis, we note that the effect of the stretching parameter on the velocity and temperature is more apparent for smaller values of the magnetic field. The variation of velocity components as well as the rate of heat transfer at the wall with the magnetic field depends on the magnitude of the stretching velocity. On the other hand, increasing the stretching velocity decreases the temperature as well as the thermal boundary layer thickness. We have found that an increase in the stretching velocity increases the velocity components but decreases the velocity boundary layer thickness. The sign of the wall shear stress was shown to depend on the stretching velocity. Finally, the effect of the heat generation/absorption parameter B on the rate of heat transfer at the wall becomes more apparent for smaller C. The results demonstrate that selecting a parameterized initial guess can be extremely useful for minimizing residual errors when used concurrently with the optimal homotopy analysis method. This approach can prove useful for a number of nonlinear differential equations arising in physics and nonlinear mechanics.

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