MORE ON KRASNER (m, n)-HYPERRINGS

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Abstract. In this paper, we introduce (idealistic) soft Krasner (m, n)-hyperrings based on the soft set theory, and derive three isomorphism theorems of soft Krasner (m, n)-hyperrings. Furthermore, we consider T-fuzzy hyperideals and falling fuzzy hyperideals of Krasner (m, n)-hyperrings by using triangular norms and the theory of falling shadows, respectively, and discuss the relationships among idealistic soft Krasner (m, n)-hyperrings, fuzzy hyperideals, T-fuzzy hyperideals and falling fuzzy hyperideals.

Keywords: Krasner (m, n)-hyperring, soft set, fuzzy set, t-norm, falling shadow.

1. Introduction

The theory of algebraic hyperstructures, introduced by Marty in 1934 [26], is a natural generalization of the theory of algebraic structures. In recent years, algebraic hyperstructures have attracted wide attention and had applications in many areas such as probabilities, fuzzy sets, rough sets, cryptography, automata, artificial intelligence, and so on [10]. n-ary hyperstructures, as a generalization of algebraic hyperstructures, introduced by Davvaz and Vougiouklis [13], were studied by many researchers [23], [42], [48]. Recently, the concept of Krasner (m, n)-ary hyperrings was defined by Mirvakili and Davvaz [27], which is a generalization of Krasner hyperrings [22]. Furthermore, Ameri and Norouzi [6] studied the prime and primary hyperideals in Krasner (m, n)-hyperrings. Also, they introduced multiplication (m, n)-hypermodules (with canonical (m, n)-hypergroups) over a given Krasner (m, n)-hyperring [7].

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After the introduction of fuzzy sets by Zadeh in 1965 [44], the theory was developed into many directions [45]. In algebraic structures, Rosenfeld [31] defined the fuzzy subgroup of a group in 1971, which initiated the study of fuzzy algebraic structures [9], [17]. In 1979, Anthony and Sherwood [8] redefined the fuzzy subgroup by using the notion of statistical triangular norms, which was proposed by Schweizer and Sklar [32] in order to generalize the ordinary triangular inequality in a metric space to the more general probabilistic metric space. Furthermore, Yuan and Lee [43] established a theoretical approach of the fuzzy algebraic systems based on the theory of falling shadows [36]. The study of connections between the fuzzy sets and algebraic hyperstructures was initiated by Zahedi et al. [46], and continued by many researchers [24], [47]. For further development, Davvaz and Corsini [12] applied the fuzzy set theory to n-ary hyperstructures and defined the concept of fuzzy n-ary subhypergroup of an n-ary hypergroup, as a generalization of the concept of fuzzy subhypergroup. As a continuation of ideas presented in [12], Davvaz [11] introduced the notion of fuzzy hyperideals of a Krasner (m, n)-hyperring and extended the fuzzy results to Krasner (m, n)-hyperrings. Moreover, Zhan et al. [48] introduced the T-fuzzy n-ary subhypergroups of an n-ary hypergroup by using triangular norms, and considered the probabilistic version of n-ary hypergroups by using random sets.

On the other hand, soft set theory, introduced by Molodtsov [28] in 1999, has been considered as an effective mathematical tool for modeling uncertainties. Different from traditional mathematical tools for dealing with uncertainties, such as probability theory, fuzzy set theory [44], vague set theory [16] and rough set theory [29], soft set theory is free from the inadequacy of the parametrization tools of these theories [28]. Molodtsov demonstrated that soft set theory has potential applications in many directions, including Riemann integration, function smoothness, game theory, measurement theory and operations research [28]. Also, soft set theory has been applied to association rules mining [18], forecasting [39], [40], mobile cloud computing [35], and decision making [14], [21, 50]. Recently, many researchers studied the algebraic structures of soft sets. Aktaş and Çağman [3], [4] defined soft groups and showed that fuzzy groups can be considered a special case of soft groups. Moreover, some basic properties of soft semirings [15], soft rings [2] and soft modules [33] were introduced. Also, Jun et al. [19], [20], [49] considered the applications of soft sets in BCK/BCI-algebras, BCH-algebras and BL-algebras. Furthermore, Yamak et al. [41] introduced soft hypergroupoids, and we presented the soft polygroups [37] and soft hypermodules [38].

In this paper, we define (idealistic) soft Krasner (m, n)-hyperrings, and investigate some related properties. Furthermore, we introduce T-fuzzy hyperideals by using triangular norms, and discuss the relationships among idealistic soft Krasner (m, n)-hyperrings, fuzzy hyperideals and T-Fuzzy hyperideals. Moreover, we give the concept of falling fuzzy hyperideals of Krasner (m, n)-hyperrings based on the theory of falling shadows, and consider the relationships among idealistic soft Krasner (m, n)-hyperrings, T-fuzzy hyperideals and falling fuzzy hyperideals.

2. Preliminaries

In this section, we review some notions and results concerning n-ary hypergroups, Krasner (m,n)-hyperrings, and soft sets.

Let H be a non-empty set and f be a mapping defined by $f : H\times H \to \mathscr{P}^*(H)$, where $\mathscr{P}^*(H)$ is the set of all non-empty subsets of H. Then f is called a binary hyperoperation on H. Let H^n denote the cartesian product $\underbrace{H \times ... \times H}_{n}$. A mapping $f : H^n \to \mathscr{P}^*(H)$ is called an n-ary hyperoperation. A non-empty set H with an n-ary hyperoperation f, denoted by (H,f), is called an n-ary hypergroupoid. If $A_1,...,A_n$ are subsets of H, then $f(A_1,...,A_n) = \cup\{f(a_1,...,a_n) \mid a_i \in A_i, i = 1,...,n\}$ [13]. For convenience, a_i^j is used to denote the sequence $a_i,a_{i+1},...,a_j$. When $j < i$, a_i^j is the empty set.

An n-ary hypergroupoid (H,f) is called an n-ary semihypergroup [13] if the following associative axiom are satisfied:

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1})$$

for every $i,j \in \{1,2,...,n\}$ and $x_1,x_2,...,x_{2n-1} \in H$. If for all $x_1^n \in H$, the set $f(x_1,x_2,..., x_n)$ is a singleton, then f is called an n-ary operation and (H,f) is called an n-ary groupoid (respectively, n-ary semigroup if the n-ary operation is associative).

An n-ary semihypergroup (H,f) in which the equation $b \in f(a_1^{i-1}, x_i, a_{i+1}^n)$ has a solution $x_i \in H$ for every $a_1^{i-1}, a_{i+1}^n, b \in H$ and $1 \leq i \leq n$, is called an n-ary hypergroup [13]. An n-ary hypergroup (H,f) is commutative, if for every permutation $\sigma \in \mathbb{S}_n$ and for all $a_1^n \in H$ we have $f(a_1,...,a_n) = f(a_{\sigma(1)},...,a_{\sigma(n)})$ [11].

A commutative n-ary hypergroup (H,f) [23] is called canonical n-ary hypergroup if

(1) there exists a unique $e \in H$, such that for all $x \in H$,

$$f(x, \underbrace{e,...,e}_{n-1}) = x;$$

(2) for all $x \in H$ there exists a unique $x^{-1} \in H$, such that

$$e \in f(x, x^{-1}, \underbrace{e,...,e}_{n-2});$$

(3) if $x \in f(x_1^n)$, then for all $1 \leq i \leq n$, we have

$$x_i \in f(x, x_1^{-1}, ..., x_{i-1}^{-1}, x_{i+1}^{-1}, ..., x_n^{-1}).$$

e is the scalar identity of (H,f) and x^{-1} is the inverse of x. The inverse of e is e.

A Krasner (m,n)-hyperring [27] is an algebraic hyperstructure (R,f,g) which satisfies the following axioms:

(1) (R, f) is a canonical m-ary hypergroup;

(2) (R, g) is an n-ary semigroup;

(3) the n-ary operation g is distributive with respect to the m-ary hyperoperation f, i.e., for every a_1^{i-1}, a_{i+1}^n, $x_1^m \in R$, $1 \leq i \leq n$,

$$g(a_1^{i-1}, f(x_1^m), a_{i+1}^n) = f(g(a_1^{i-1}, x_1, a_{i+1}^n), ..., g(a_1^{i-1}, x_m, a_{i+1}^n));$$

(4) 0 is a zero element (absorbing element) of the n-ary operation g, i.e., for every $x_2^n \in R$ we have

$$g(0, x_2^n) = g(x_2, 0, x_3^n) = ... = g(x_2^n, 0) = 0.$$

It is clear that every Krasner hyperring [22] is a Krasner (2, 2)-hyperring. Also, every Krasner $(m, 0)$-hyperring is a canonical m-ary hypergroup and every Krasner $(0, n)$-hyperring is an n-ary semigroup. Let S be a non-empty subset of a Krasner (m, n)-hyperring (R, f, g). If (S, f, g) is a Krasner (m, n)-hyperring, then S called a subhyperring of R.

Let I be a non-empty subset of a Krasner (m, n)-hyperring R and $1 \leq i \leq n$. We call I an i-hyperideal of R [27] if

(1) I is a subhypergroup of the canonical m-ary hypergroup (R, f), i.e., (I, f) is a canonical m-ary hypergroup;

(2) for every $x_1^n \in R$, $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$.

If for every $1 \leq i \leq n$, I is an i-hyperideal, then I called a hyperideal of R. Clearly, every hyperideal of R is a subhyperring of R. Furthermore, we have that a non-empty subset I of a Krasner (m, n)-hyperring R is a hyperideal if

(1) for every $x_1^m \in I$, $f(x_1^m) \subseteq I$;

(2) for every $x \in I$, $-x \in I$;

(3) for every $x_1^n \in R$ and $1 \leq i \leq n$, $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$.

A hyperideal I of a Krasner (m, n)-hyperring R is called normal if and only if for every $r \in R$, $f(-r, I, r, \overset{(m-3)}{0}) \subseteq I$. Suppose that I is a normal hyperideal of R, then the relation I^* defined by

$$xI^*y \text{ if and only if } f(x, -y, \overset{(m-2)}{0}) \cap I \neq \emptyset, \forall x, y \in R$$

is an equivalence relation on R [27]. Let $I^*[x]$ be the equivalence class of the element $x \in R$, then $I^*[x] = f(I, x, \overset{(m-2)}{0})$. Also, we have $I^*[(f(a_1^m))] = I^*[a]$ for all $a \in f(a_1^m)$ and $a_1^m \in R$. The set of all equivalence classes $[R : I^*] = \{I^*[x] | x \in R\}$

is a Krasner (m,n)-hyperring with the m-ary hyperoperation f/I and the n-ary operation g/I defined by

$$f/I(I^*[x_1],...,I^*[x_m]) = \{I^*[z] \mid z \in f(I^*[x_1],...,I^*[x_m])\}, \forall x_1^m \in R,$$

$$g/I(I^*[x_1],...,I^*[x_n]) = I^*[g(x_1^n)], \forall x_1^n \in R.$$

Let (R_1, f_1, g_1) and (R_2, f_2, g_2) be two Krasner (m,n)-hyperrings. A mapping $\varphi : R_1 \to R_2$ is called a homomorphism if for all $x_1^m \in R_1$ and $y_1^n \in R_1$, $\varphi(f_1(x_1,...,x_m)) = f_2(\varphi(x_1),...,\varphi(x_m))$, and $\varphi(g_1(y_1,...,y_n)) = g_2(\varphi(y_1),...,\varphi(y_n))$. A homomorphism φ is an isomorphism if φ is injective and surjective. $R_1 \cong R_2$ is used to denote that R_1 is isomorphic to R_2 [27].

Let $\varphi : R_1 \to R_2$ be a homomorphism, then (1) $\varphi(0_{R_1}) = 0_{R_2}$; (2) for all $x \in R$, $\varphi(-x) = -\varphi(x)$; (3) let $ker\varphi = \{x \in R_1 | \varphi(x) = 0_{R_2}\}$, then φ is injective if and only if $ker\varphi = \{0_{R_1}\}$.

Now, we review some notions about soft sets. See [5], [15], [20], [25], [28] for the following definitions. Let U be an initial universe set and E be a set of parameters. $\mathscr{P}(U)$ denotes the power set of U and $A \subseteq E$.

A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathscr{P}(U)$. In fact, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). There are several examples in [25]. For a soft set (F, A) over U, the set $\mathrm{Supp}(F, A) = \{x \in A \mid F(x) \neq \varnothing\}$ is called the support of the soft set (F, A). A soft set (F, A) is non-null if $\mathrm{Supp}(F, A) \neq \varnothing$. For two soft sets (F, A) and (G, B) over U, we say that (F, A) is a soft subset of (G, B), denoted by $(F, A)\widetilde{\subseteq}(G, B)$, if $A \subseteq B$ and for all $e \in A$, $F(e) \subseteq G(e)$. Two soft sets (F, A) and (G, B) over U are called soft equal if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$.

Let $(F_i, A_i)_{i\in\Lambda}$ be a non-empty family of soft sets over U, then

(1) the extended intersection (union) of $(F_i, A_i)_{i\in\Lambda}$ is the soft set $(G, B) = (\bigcap_{\mathscr{E}})_{i\in\Lambda}(F_i, A_i)$ $((\bigcup_{\mathscr{E}})_{i\in\Lambda}(F_i, A_i))$, where $B = \bigcup_{i\in\Lambda} A_i$, and for all $e \in B$, $G(e) = \bigcap_{i\in\Lambda(e)} F_i(e)$ $(\bigcup_{i\in\Lambda(e)} F_i(e))$ and $\Lambda(e) = \{i \in \Lambda \mid e \in A_i\}$;

(2) the restricted intersection (union) of $(F_i, A_i)_{i\in\Lambda}$ is the soft set $(G, B) = (\bigcap_{\mathscr{R}})_{i\in\Lambda}(F_i, A_i)$ $((\bigcup_{\mathscr{R}})_{i\in\Lambda}(F_i, A_i))$, where $B = \bigcap_{i\in\Lambda} A_i \neq \varnothing$ and for all $e \in B$, $G(e) = \bigcap_{i\in\Lambda} F_i(e)$ $(\bigcup_{i\in\Lambda} F_i(e))$;

(3) the $\wedge$-intersection ($\vee$-union) of $(F_i, A_i)_{i\in\Lambda}$ is the soft set $(G, B) = \widetilde{\bigwedge}_{i\in\Lambda}(F_i, A_i)$ $(\widetilde{\bigvee}_{i\in\Lambda}(F_i, A_i))$, where $B = \prod_{i\in\Lambda} A_i$ and for all $e = (e_i)_{i\in\Lambda} \in B$, $G(e) = \bigcap_{i\in\Lambda} F_i(e_i)$ $(\bigcup_{i\in\Lambda} F_i(e_i))$;

(4) the cartesian product of $(F_i, A_i)_{i\in\Lambda}$ is the soft set $(G, B) = \prod_{i\in\Lambda}(F_i, A_i)$, where $B = \prod_{i\in\Lambda} A_i$ and for all $e = (e_i)_{i\in\Lambda} \in B$, $G(e) = \prod_{i\in\Lambda} F_i(e_i)$.

3. Soft Krasner (m, n)-hyperrings

In this section, we firstly introduce the concepts of soft Krasner (m, n)-hyperring and idealistic soft Krasner (m, n)-hyperring. Then, we consider isomorphism theorems of soft Krasner (m, n)-hyperrings. In what follows, let R be a Krasner (m, n)-hyperring.

Definition 3.1. Let (F, A) be a non-null soft set over R. Then (F, A) is called an (idealistic) soft Krasner (m, n)-hyperring over R if $F(x)$ is a subhyperring (hyperideal) of R for all $x \in \mathrm{Supp}(F, A)$.

Obviously, every idealistic soft Krasner (m, n)-hyperring over R is a soft Krasner (m, n)-hyperring over R. Let (F, A) be an idealistic soft Krasner (m, n)-hyperring over R, then (F, A) is called a trivial (whole) idealistic soft Krasner (m, n)-hyperring over R if $F(x) = \{0\}$ $(F(x) = R)$ for all $x \in A$.

Example 3.2. Let $R = \{0, 1, 2, 3\}$ be a set with a 2-ary hyperoperation $+$ and an n-ary operation g defined by

$+$	0	1	2	3
0	0	1	2	3
1	1	$\{0,1\}$	3	$\{2,3\}$
2	2	3	0	1
3	3	$\{2,3\}$	1	$\{0,1\}$

$$g(x_1^n) = \begin{cases} 2 & \text{if } x_1^n \in \{2,3\}, \\ 0 & \text{else.} \end{cases}$$

It follows that $(R, +, g)$ is a Krasner $(2, n)$-hyperring [27]. Let $\mathbb{N}$ be the set of natural numbers and (F, A) be the soft set over R, where $A = \mathbb{N}$ and $F : A \to \mathscr{P}(R)$ is the set-valued function defined by

$$F(x) = \begin{cases} R & \text{if } 2 \mid x, \\ \{0,2\} & \text{otherwise} \end{cases}$$

for all $x \in A$, then (F, A) is an idealistic soft Krasner $(2, n)$-hyperring over R. Also, (F, A) is a soft Krasner $(2, n)$-hyperring over R.

Theorem 3.3. *Let $(F_i, A_i)_{i\in\Lambda}$ be a non-empty family of (idealistic) soft Krasner (m, n)-hyperrings over R, then*

(1) *the extended intersection $(\bigcap_{\mathscr{E}})_{i\in\Lambda}(F_i, A_i)$ is an (idealistic) soft Krasner (m, n)-hyperring over R if it is non-null;*

(2) *the restricted intersection $(\bigcap_{\mathscr{R}})_{i\in\Lambda}(F_i, A_i)$ is an (idealistic) soft Krasner (m, n)-hyperring over R if it is non-null;*

(3) *if* $A_i \cap A_j = \emptyset$ *for all* $i, j \in \Lambda$ *and* $i \neq j$*, then the extended union* $(\bigcup_{\mathscr{E}})_{i\in\Lambda}(F_i, A_i)$ *is an (idealistic) soft Krasner* (m,n)*-hyperring over* R*;*

(4) *the* $\wedge$*-intersection* $\widetilde{\bigwedge}_{i\in\Lambda}(F_i, A_i)$ *is an (idealistic) soft Krasner* (m,n)*-hyperring over* R*;*

(5) *the cartesian product* $\prod_{i\in\Lambda}(F_i, A_i)$ *is an (idealistic) soft Krasner* (m,n)*-hyperring over* R*.*

Proof. We only prove (1), and the other proofs are similar. We can write $(\bigcap_{\mathscr{E}})_{i\in\Lambda}(F_i, A_i) = (G, B)$, where $B = \bigcup_{i\in\Lambda} A_i$ and $G(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$ for all $x \in B$. For all $x \in \mathrm{supp}(G, B)$, $G(x) = \bigcap_{i\in\Lambda(x)} F_i(x) \neq \emptyset$, that is, $F_i(x) \neq \emptyset$ for all $i \in \Lambda(x)$. Also, $F_i(x)$ is a subhyperring (hyperideal) of R, since $(F_i, A_i)_{i\in\Lambda}$ be a non-empty family of (idealistic) soft Krasner (m,n)-hyperrings over R. Therefore, we have that $G(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$ is a subhyperring (hyperideal) of R for all $x \in \mathrm{supp}(G, B)$. So $(\bigcap_{\mathscr{E}})_{i\in\Lambda}(F_i, A_i) = (G, B)$ is an (idealistic) soft Krasner (m,n)-hyperring over R. ∎

Definition 3.4. Let R_1 and R_2 be two Krasner (m,n)-hyperrings, (F, A) and (G, B) be soft Krasner (m,n)-hyperrings over R_1 and R_2, respectively, and $\varphi : R_1 \to R_2$ and $\psi : A \to B$ be two mappings. If φ is a homomorphism, and $\varphi(F(x)) = G(\psi(x))$ for all $x \in A$, then (φ, ψ) is called a soft homomorphism, and (F, A) is soft homomorphic to (G, B), denoted by $(F, A) \sim (G, B)$.

In Definition 3.4, if φ is a monomorphism (resp. epimorphism, isomorphism) and ψ is an injective (resp. surjective, bijective) mapping, then (φ, ψ) is called a soft monomorphism (resp. epimorphism, isomorphism), and (F, A) is soft monomorphic (resp. epimorphic, isomorphic) to (G, B). $(F, A) \simeq (G, B)$ is used to denote that (F, A) is soft isomorphic to (G, B).

Example 3.5. Let R be the Krasner $(2,n)$-hyperring and (F, A) be the soft Krasner $(2,n)$-hyperring over R described in Example 3.2. Let (G, B) be the soft set over R, where $B = \mathbb{N}$ and $G : B \to \mathscr{P}(R)$ is the set-valued function defined by

$$G(x) = \begin{cases} \{0, 2\} & \text{if } 2 \mid x, \\ \emptyset & \text{otherwise} \end{cases}$$

for all $x \in B$, then (G, B) is a soft Krasner $(2,n)$-hyperring over R.

Let $\varphi : R \to R$ be the mapping defined by

$$\varphi(x) = \begin{cases} 2 & \text{if } x \in \{2, 3\}, \\ 0 & \text{otherwise} \end{cases}$$

for all $x \in R$, then φ is a homomorphism. Define the mapping $\psi : A \to B$ by $\psi(x) = 2x$ for all $x \in A$. It is easy to check that $\varphi(F(x)) = G(\psi(x))$ for all $x \in A$. Therefore, (φ, ψ) is a soft homomorphism and $(F, A) \sim (G, B)$.

Next, we establish three isomorphism theorems of soft Krasner (m,n)-hyperrings. In the following theorems, we write $([F : I^*], A)$, which means that $[F : I^*](x) = [F(x) : I^*]$ for all $x \in A$, $I \subseteq F(x)$ for $x \in$Supp(F,A) and $[F : I^*](x) = \emptyset$ for $x \in A-$Supp(F,A), where (F,A) is a soft Krasner (m,n)-hyperring over R, and I is a normal hyperideal of R.

Theorem 3.6. (First Isomorphism Theorem) *Let (F,A) and (G,B) be soft Krasner (m,n)-hyperrings over Krasner (m,n)-hyperrings R_1 and R_2, respectively. If (φ,ψ) is a soft epimorphism from (F,A) to (G,B) such that $I = ker\varphi$ is a normal hyperideal of R_1 and $I \subseteq F(x)$ for $x \in$Supp(F,A), then*

(1) *$([F : I^*], A) \simeq (\varphi(F), A)$;*

(2) *if ψ is bijective, then $([F : I^*], A) \simeq (G,B)$,*

where $\varphi(F)(x) = \varphi(F(x))$ for all $x \in A$.

Proof. (1) Clearly, $([F : I^*], A)$ and $(\varphi(F), A)$ are soft Krasner (m,n)-hyperrings over $[R_1 : I^*]$ and R_2, respectively. Define $\overline{\varphi} : [R_1 : I^*] \to R_2$ by $\overline{\varphi}(I^*[x]) = \varphi(x)$, for all $x \in R_1$. We show that $\overline{\varphi}$ is well-defined. Suppose that xI^*y, then we have $f_1(x,-y,\overset{(m-2)}{0_{R_1}}) \cap I \neq \emptyset$, that is, there exists $z \in f_1(x,-y,\overset{(m-2)}{0_{R_1}}) \cap I$. Thus, we have $\varphi(z) = 0_{R_2}$ and $\varphi(z) \in \varphi(f_1(x,-y,\overset{(m-2)}{0_{R_1}}))$, which imply that $0_{R_2} \in f_2(\varphi(x),-\varphi(y),\overset{(m-2)}{0_{R_2}}))$, i.e., $\varphi(x) = \varphi(y)$. We can check easily that $\overline{\varphi}$ is surjective, since φ is surjective. If $\varphi(x) = \varphi(y)$, we have $0_{R_2} \in f_2(\varphi(x),-\varphi(y),\overset{(m-2)}{0_{R_2}})$, which implies that there exists $z \in f_1(x,-y,\overset{(m-2)}{0_{R_1}})$ such that $z \in ker\varphi$, i.e., $f_1(x,-y,\overset{(m-2)}{0_{R_1}}) \cap I \neq \emptyset$. It follows that $I^*[x] = I^*[y]$. So $\overline{\varphi}$ is injective. Also, we have

$$\begin{aligned}
\overline{\varphi}(f_1/I(I^*[x_1],...,I^*[x_m])) &= \overline{\varphi}(\{I^*[z] | z \in f_1(I^*[x_1],...,I^*[x_m])\}) \\
&= \overline{\varphi}(\{I^*[z] | z \in f_1(I, f_1(x_1^m), \overset{(m-2)}{0_{R_1}})\}) \\
&= \varphi(f_1(I, f_1(x_1^m), \overset{(m-2)}{0_{R_1}})) \\
&= f_2(\varphi(I), \varphi(f_1(x_1^m)), \overset{(m-2)}{\varphi(0_{R_1})}) \\
&= f_2(0_{R_2}, f_2(\varphi(x_1),...,\varphi(x_m)), \overset{(m-2)}{0_{R_2}}) \\
&= f_2(\varphi(x_1),...,\varphi(x_m)) \\
&= f_2(\overline{\varphi}(I^*[x_1]),...,\overline{\varphi}(I^*[x_m])),
\end{aligned}$$

$$\begin{aligned}
\overline{\varphi}(g_1/I(I^*[x_1],...,I^*[x_n])) &= \overline{\varphi}(\{I^*[g_1(I^*[x_1],...,I^*[x_n])]\}) = \varphi(g_1(I^*[x_1],...,I^*[x_n])) \\
&= g_2(\varphi(I^*[x_1]),...,\varphi(I^*[x_n])) \\
&= g_2(f_2(0_{R_2},\varphi(x_1),\overset{(m-2)}{0_{R_2}}),...,f_2(0_{R_2},\varphi(x_n),\overset{(m-2)}{0_{R_2}})) \\
&= g_2(\varphi(x_1),...,\varphi(x_n)) = g_2(\overline{\varphi}(I^*[x_1]),...,\overline{\varphi}(I^*[x_n])).
\end{aligned}$$

So $\overline{\varphi}$ is an isomorphism. Next, we define $\overline{\psi} : A \to A$ by $\overline{\psi}(x) = x$ for all $x \in A$. Clearly, $\overline{\psi}$ is a bijective mapping. For all $x \in A$, we have $\overline{\varphi}([F(x) : I^*]) = \varphi(F(x)) = \varphi(F(\psi(x))) = \varphi(F)(\psi(x))$. It follows that $(\overline{\varphi}, \overline{\psi})$ is a soft isomorphism and $([F : I^*], A) \simeq (\varphi(F), A)$.

(2) From the proof above, we have $\overline{\varphi}$ is an isomorphism. ψ is bijective and $\overline{\varphi}([F(x) : I^*]) = \varphi(F(x)) = G(\psi(x))$ for all $x \in A$. Therefore, $(\overline{\varphi}, \psi)$ is a soft isomorphism, and $([F : I^*], A) \simeq (G, B)$. ∎

Theorem 3.7. (Second Isomorphism Theorem) *Let I_1^m be hyperideals of R, and there exists $1 \leq j \leq m$ such that I_j is a normal hyperideal of R. If (F, A) is a soft Krasner (m,n)-hyperring over $f(I_1^{j-1}, 0, I_{j+1}^m)$, then*

$$([F : (f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j)^*], A) \simeq ([F_{I_j/0} : I_j^*], A),$$

where for all $x \in A$, $F_{I_j/0}(x) = f(\overline{I}_1^{j-1}, I_j, \overline{I}_{j+1}^m)$ if we assume that $F(x) = f(\overline{I}_1^{j-1}, 0, \overline{I}_{j+1}^m)$, $\overline{I}_k \subseteq I_k$, $k \in \{1, ..., j-1, j+1, ..., m\}$, and $f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j \subseteq F(x)$ for $x \in \mathrm{Supp}(F, A)$.

Proof. It is easy to deduce that $([F : (f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j)^*], A)$ and $([F_{I_j/0} : I_j^*], A)$ are soft hyperideals over $[f(I_1^{j-1}, 0, I_{j+1}^m) : (f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j)^*]$ and $[f(I_1^m) : I_j^*]$, respectively. We define $\varphi : f(I_1^{j-1}, 0, I_{j+1}^m) \to [f(I_1^m) : I_j^*]$ by $\varphi(x) = I_j^*[x]$ for all $x \in f(I_1^{j-1}, 0, I_{j+1}^m)$. Clearly, φ is a homomorphism. For any $I_j^*[y] \in [f(I_1^m) : I_j^*]$, where $y \in f(I_1^m)$, i.e., there exist $a_k \in I_k$, $k \in \{1, ..., m\}$ such that $y \in f(a_1^m)$, we have $I_j^*[y] = I_j^*[f(a_1^m)] = f(I_j, f(a_1^m), \overset{(m-2)}{0}) = f(a_1^{j-1}, f(I_j, a_j, \overset{(m-2)}{0}), a_{j+1}^m) = f(a_1^{j-1}, I_j, a_{j+1}^m) = f(I_j, f(a_1^{j-1}, 0, a_{j+1}^m), \overset{(m-2)}{0}) = I_j^*[x] = \varphi(x), x \in f(a_1^{j-1}, 0, a_{j+1}^m) \subseteq f(I_1^{j-1}, 0, I_{j+1}^m)$. It follows that φ is an epimorphism. Now, we define $\psi : A \to A$ by $\psi(x) = x$ for all $x \in A$. ψ is a bijective mapping. For all $x \in A$, we have $\varphi(F(x)) = \{I_j^*[a] \mid a \in F(x)\} = [F_{I_j/0} : I_j^*](x) = [F_{I_j/0} : I_j^*](\psi(x))$.

Next, we give the proof of $\{I_j^*[a] \mid a \in F(x)\} = [F_{I_j/0} : I_j^*](x)$. Clearly, $\{I_j^*[a] \mid a \in F(x)\} \subseteq [F_{I_j/0} : I_j^*](x)$. For any $I_j^*[b] \in [F_{I_j/0} : I_j^*](x)$, where $b \in F_{I_j/0}(x)$, which implies that there exist $a_k \in \overline{I}_k \subseteq I_k$, $k \in \{1, ..., j-1, j+1, ..., m\}$ and $a_j \in I_j$ such that $b \in f(a_1^m)$, we have $I_j^*[b] = I_j^*[f(a_1^m)] = f(I_j, f(a_1^m), \overset{(m-2)}{0}) = f(a_1^{j-1}, f(I_j, a_j, \overset{(m-2)}{0}), a_{j+1}^m) = f(a_1^{j-1}, I_j, a_{j+1}^m) = f(I_j, f(a_1^{j-1}, 0, a_{j+1}^m), \overset{(m-2)}{0}) = I_j^*[t] (t \in f(a_1^{j-1}, 0, a_{j+1}^m) \subseteq F(x)) \in \{I_j^*[a] \mid a \in F(x)\}$. Thus, (φ, ψ) is a soft epimorphism from (F, A) to $([F_{I_j/0} : I_j^*], A)$. Because $f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j$ is a normal hyperideal of $f(I_1^{j-1}, 0, I_{j+1}^m)$, if we have $ker\varphi = f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j$, then $([F : (f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j)^*], A) \simeq ([F_{I_j/0} : I_j^*], A)$. For any $x \in f(I_1^{j-1}, 0, I_{j+1}^m)$, $x \in ker\varphi \Leftrightarrow \varphi(x) = I_j^*[0] = I_j \Leftrightarrow I_j^*[x] = f(I_j, x, \overset{(m-2)}{0_{R_2}}) = I_j \Leftrightarrow x \in I_j$ (since $x \in f(I_1^{j-1}, 0, I_{j+1}^m)$) $\Leftrightarrow x \in I_j \cap f(I_1^{j-1}, 0, I_{j+1}^m)$. So $ker\varphi = f(I_1^{j-1}, 0, I_{j+1}^m) \cap I_j$. ∎

Theorem 3.8. (Third Isomorphism Theorem) *Let I and N be normal hyperideals of R such that $I \subseteq N$. If (F, A) is a soft Krasner (m, n)-hyperring over R, and $N \subseteq F(x)$ for all $x \in$ supp(F, A), then* $([[F : I^*] : [N : I^*]^*], A) \simeq ([F : N^*], A)$.

Proof. We can obtain that $[N : I^*]$ is a normal hyperideal of $[R : I^*]$. I and N are normal hyperideals of R, and $I \subseteq N$, then $[[R : I^*] : [N : I^*]^*]$ is defined. Also, it is easy to obtain that $([F : I^*], A)$, $([F : N^*], A)$ and $([[F : I^*] : [N : I^*]^*], A)$ are soft Krasner (m, n)-hyperrings over $[R : I^*]$, $[R : N^*]$ and $[[R : I^*] : [N : I^*]^*]$, respectively. $\varphi : [R : I^*] \to [R : N^*]$, defined by $\varphi(I^*[x]) = N^*[x]$, is an epimorphism, and $\psi : A \to A$, defined by $\psi(x) = x$ for all $x \in A$, is a bijective mapping. Also, we have $\varphi([F(x) : I^*]) = [F(x) : N^*] = [F(\psi(x)) : N^*]$, for all $x \in A$. Therefore, (φ, ψ) is a soft epimorphism from $([F : I^*], A)$ to $([F : N^*], A)$. According to the first isomorphism theorem, if $ker\varphi = [N : I^*]$, then $([[F : I^*] : [N : I^*]^*], A) \simeq ([F : N^*], A)$. For any $I^*[x] \in [R : I^*]$, $I^*[x] \in ker\varphi \Leftrightarrow \varphi(I^*[x]) = N^*[0] = N \Leftrightarrow N^*[x] = f(N, x, \overset{(m-2)}{0}) = N \Leftrightarrow x \in N \Leftrightarrow I^*[x] \in [N : I^*]$. It follows that $ker\varphi = [N : I^*]$. ∎

4. The relationships among idealistic soft Krasner (m, n)-hyperrings, fuzzy hyperideals and T-Fuzzy hyperideals

Let μ be a fuzzy subset of R, and $1 \leq i \leq n$. Then μ is called a fuzzy i-hyperideal of R if μ is a fuzzy subhypergroup of the canonical m-ary hypergroup (R, f), and $\mu(x_i) \leq \mu(g(x_1^n))$, for all $x_1^n \in R$. If for every $1 \leq i \leq n$, μ is a fuzzy i-hyperideal, then μ is called a fuzzy hyperideal of R [11]. Therefore, there is the following definition.

Definition 4.1([11]). A fuzzy subset μ of R is called a fuzzy hyperideal of R if the following conditions hold:

(1) $\min\{\mu(x_1), ..., \mu(x_m)\} \leq \inf_{z \in f(x_1^m)} \{\mu(z)\}$, for all $x_1^m \in R$;

(2) $\mu(x) \leq \mu(-x)$, for all $x \in R$;

(3) $\max\{\mu(x_1), ..., \mu(x_n)\} \leq \mu(g(x_1^n))$, for all $x_1^n \in R$.

A triangular norm (t-norm) [8] is a function $T : [0, 1] \times [0, 1] \to [0, 1]$, such that for all $x, y, z \in [0, 1]$ the following axioms are satisfied:

(1) $T(x, T(y, z)) = T(T(x, y), z)$;

(2) $T(x, y) = T(y, x)$;

(3) $T(x, y) \leq T(x, z)$ if $y \leq z$;

(4) $T(x, 1) = x$.

Obviously, the function min, defined by $\min(a,b) = \min\{a,b\}$ for every $a, b \in [0,1]$, is a t-norm. There are many other t-norms. $T_{Łuk}$ and T_{Prod} are two t-norms used frequently, defined by $T_{Łuk}(a,b) = \max\{a+b-1, 0\}$ and $T_{Prod}(a,b) = ab$, respectively, for all $a, b \in [0,1]$. Every t-norm T has the following property: $T(a,b) \leq \min\{a,b\}$ for all $a, b \in [0,1]$. Let T be a t-norm. Then the fuzzy set μ of a non-empty set X satisfies the imaginable property if $Im(\mu) \subseteq \Delta_T$, where $\Delta_T = \{\alpha \in [0,1] \mid T(\alpha,\alpha) = \alpha\}$ [30].

Definition 4.2. Let μ be a fuzzy subset of R. Then μ is called a T-fuzzy hyperideal of R if the following axioms hold:

(1) $T_m(\mu(x_1), ..., \mu(x_m)) \leq \inf\limits_{z \in f(x_1^m)} \{\mu(z)\}$, for all $x_1^m \in R$;

(2) $\mu(x) \leq \mu(-x)$, for all $x \in R$;

(3) $\max\{\mu(x_1), ..., \mu(x_m)\} \leq \mu(g(x_1^n))$, for all $x_1^n \in R$,

where T is a t-norm and for $a_1, ..., a_m \in [0,1]$, if $m > 2$, $T_m(a_1, ..., a_m) = T(a_i, T_{m-1}(a_1, ..., a_{i-1}, a_{i+1}, ..., a_m))$; $T_2 = T$; $T_1 = id$(identity)[1]. For convenience, we write T_m by T.

Example 4.3. Consider the Krasner $(2,n)$-hyperring described in Example 3.2. We define a fuzzy set μ in R by $\mu(0) = \mu(1) = 0.8$ and $\mu(2) = \mu(3) = 0.6$. Then μ is a $T_{Łuk}$-fuzzy hyperideal of R.

For a fuzzy point x_t and a fuzzy set μ in a non-empty set X, we say $x_t \in \mu$, which means that $\mu(x) \geq t$. Let $A \subseteq [0,1]$. Consider the set-valued function $F : A \to \mathscr{P}(X)$, defined by $t \mapsto \mu_t = \{x \in X \mid x_t \in \mu\}$, then (F, A) is called a $\in$-soft set over X [49].

Theorem 4.4. *Let μ be a fuzzy set of R, and (F, A) be a $\in$-soft set over R with A=[0,1]. Then μ is a fuzzy hyperideal of R if and only if (F, A) is an idealistic soft Krasner (m,n)-hyperring over R.*

Proof. Assume that μ is a fuzzy hyperideal of R. Let $t \in \mathrm{Supp}(F, A)$ and $x_1^m, x \in F(t)$. Then we have $(x_i)_t \in \mu$ for all $1 \leq i \leq m$ and $x_t \in \mu$. Since $t \leq \min\{\mu(x_1), ..., \mu(x_m)\} \leq \inf\limits_{z \in f(x_1^m)} \{\mu(z)\}$ and $t \leq \mu(x) \leq \mu(-x)$, we obtain $f(x_1^m) \subseteq F(t)$ and $-x \in F(t)$. If $x_1^{i-1}, x_{i+1}^n \in R$ and $x_i \in F(t)$, then $(x_i)_t \in \mu$. So we have $t \leq \mu(x_i) \leq \max\{\mu(x_1), ..., \mu(x_n)\} \leq \mu(g(x_1^n))$, which implies that $\mu(g(x_1^n)) \in F(t)$, i.e., $\mu(g(x_1, F(t), x^n)) \subseteq F(t)$. Hence (F, A) is an idealistic soft Krasner (m,n)-hyperring over R.

Conversely, let (F, A) be an idealistic soft Krasner (m,n)-hyperring over R. For all $x_1^m \in R$, if there exists $z \in f(x_1^m)$ such that $\mu(z) < \min\{\mu(x_1), ..., \mu(x_m)\}$, then choose $t \in A$ such that $\mu(z) < t \leq \min\{\mu(x_1), ..., \mu(x_m)\}$. It follows that $(x_i)_t \in \mu$ for all $1 \leq i \leq m$, but $z_t \overline{\in} \mu$. This contradicts to that $F(t)$ is a hyperideal of R. So we have $\min\{\mu(x_1),...,\mu(x_m)\} \leq \inf\limits_{z\in f(x_1^m)}\{\mu(z)\}$. For all $x \in R$, assume that $\mu(-x) < \mu(x)$, then there is $r \in A$ such that $\mu(-x) < r \leq \mu(x)$. Thus, we have $(x)_r \in \mu$, but $(-x)_r \overline{\in} \mu$, which is a contradiction. So we have $\mu(x) \leq \mu(-x)$ for all $x \in R$. Let $x_1^n \in R$. Suppose that $s = \mu(x_i) = \max\{\mu(x_1),...,\mu(x_n)\}$, then $x_i \in F(s)$. Since $F(s)$ is a hyperideal of R, $g(x_1^n) \in F(s)$, i.e., $\mu(g(x_1^n)) \geq s$, which implies that $\max\{\mu(x_1),...,\mu(x_n\} \leq \mu(g(x_1^n))$. Therefore, μ is a fuzzy hyperideal of R. ∎

Theorem 4.5. *Let μ be an imaginable T-fuzzy hyperideal of R, then μ is a fuzzy hyperideal of R.*

Proof. Since μ is an imaginable T-fuzzy hyperideal of R, for all $x_1^m \in R$, we have $T(\mu(x_1),...,\mu(x_m)) \leq \inf_{z\in f(x_1^m)} \mu(z)$. Furthermore, we obtain $\min\{\mu(x_1),...,\mu(x_m)\} = T(\min\{\mu(x_1),...,\mu(x_m)\},...,\min\{\mu(x_1),...,\mu(x_m)\}) \leq T(\mu(x_1),...,\mu(x_m)) \leq \min(\mu(x_1),...,\mu(x_m))$. That is, $T(\mu(x_1),...,\mu(x_m)) = \min(\mu(x_1),...,\mu(x_m))$. It follows that $\min(\mu(x_1),...,\mu(x_m)) \leq \inf_{z\in f(x_1^m)} \mu(z)$. Therefore, μ is a fuzzy hyperideal of R. ∎

Theorem 4.6. *Let μ be an imaginable fuzzy subset of R and (F, A) be a $\in$-soft set over R with A=[0,1]. Then μ is a T-fuzzy hyperideal of R if and only if (F, A) is an idealistic soft Krasner (m, n)-hyperring over R.*

Proof. Assume that μ is a T-fuzzy hyperideal of R. According to Theorem 4.5, we have that μ is a fuzzy hyperideal of R. Furthermore, by using Theorem 4.4, (F, A) is an idealistic soft Krasner (m, n)-hyperring over R. Conversely, if (F, A) is an idealistic soft Krasner (m, n)-hyperring over R, then μ is a fuzzy hyperideal of R by Therorem 4.4. It follows that $\inf_{z\in f(x_1^m)} \mu(z) \geq \min\{\mu(x_1),...,\mu(x_m)\} \geq T(\mu(x_1),...,\mu(x_m))$ for all $x_1^m \in R$. Therefore, we have μ is a T-fuzzy hyperideal of R. ∎

Theorem 4.7. *Let I be a hyperideal of R, and μ be a fuzzy subset of R given by for all $s, t \in [0, 1]$ with $s > t$, $\mu(x) = s$, if $x \in I$; otherwise, $\mu(x) = t$. Then μ is a $T_{Łuk}$-fuzzy hyperideal of R. If $s = 1$ and $t = 0$ then μ is imaginable.*

Proof. Let $x_1^m \in R$. If $x_1^m \in I$ then $f(x_1^m) \subseteq I$ since I is a hyperideal of R. Thus, we have $T_{Łuk}(\mu(x_1),...,\mu(x_m)) = \max\{ms - (m-1), 0\} \leq s = \inf_{z\in f(x_1^m)} \mu(z)$. Otherwise, assume that $x_{\sigma(1)},...,x_{\sigma(i)} \in I$ and $x_{\sigma(i+1)},...,x_{\sigma(m)} \notin I$, where $\sigma \in \mathbb{S}_n$ is a permutation, then $T_{Łuk}(\mu(x_1),...,\mu(x_m)) = \max\{is+(m-i)t-(m-1), 0\} \leq t = \inf_{z\in f(x_1^m)} \mu(z)$. Let $x \in R$. If $x \in I$, then $-x \in I$. So we have $\mu(-x) = s = \mu(-x)$. If $x \notin I$, then $\mu(-x) \geq t = \mu(x)$. Let $x_1^n \in R$. If there exists $x_i \in I$, $i \in \{0,...,n\}$, then $g(x_1^n) \in I$ since I is a hyperideal of R. Hence $\max\{\mu(x_1),...,\mu(x_n)\} = s = \mu(g(x_1^n))$. Otherwise, $\max\{\mu(x_1),...,\mu(x_n)\} = t \leq \mu(g(x_1^n))$. Consequently, μ is a $T_{Łuk}$-fuzzy hyperideal of R. When $s = 1$ and $t = 0$, it is clear that μ is imaginable. ∎

Theorem 4.8. *Let μ be a fuzzy set of R with $Im(\mu) = \{\lambda_0, \lambda_1, ..., \lambda_t\}$, where $\lambda_i < \lambda_j$ whenever $i > j$. If there exists a chain of hyperideal of R: $I_0 \subseteq I_1 \subseteq ... \subseteq I_t = R$ such that $\mu(I'_k) = \lambda_k$, where $I'_k = I_k \backslash I_{k-1}, I_{-1} = \emptyset$ for $k = 0, 1, ..., t$, then μ is a T-fuzzy hyperideal of R.*

Proof. For all $x_1^m \in R$, assume that $x_1 \in I'_{\alpha_1}, ..., x_m \in I'_{\alpha_m}$, where $\alpha_1, ..., \alpha_m \in \{0, 1, ..., t\}$. Then $\mu(x_1) = \lambda_{\alpha_1}, ..., \mu(x_m) = \lambda_{\alpha_m}$. Set $\lambda_{\alpha_c} = \min\{\mu(x_1), ..., \mu(x_m)\}$, then we have $f(x_1^m) \subseteq I_{\alpha_c}$. It follows that

$$\inf_{z \in f(x_1^m)} \mu(z) \geq \lambda_{\alpha_c} = \min\{\mu(x_1), ..., \mu(x_m)\} \geq T(\mu(x_1), ..., \mu(x_m)).$$

For all $x \in R$, we have $x \in I'_k$ for some $k = \{0, 1, ..., t\}$. It is clear that $-x \in I'_k$. So we have $\mu(-x) = \lambda_k = \mu(x)$. Now, for all $x_1^n \in R$, suppose that $x_1 \in I'_{\beta_1}, ..., x_n \in I'_{\beta_n}$, where $\beta_1, ..., \beta_n \in \{0, 1, ..., t\}$. Then $\mu(x_1) = \lambda_{\beta_1}, ..., \mu(x_n) = \lambda_{\beta_n}$. Set $\lambda_{\beta_c} = \mu(x_c) = \max\{\mu(x_1), ..., \mu(x_n)\}$. Since I_{β_c} is a hyperideal of R, we have $g(x_1^n) \in I_{\beta_c}$. Therefore, we have $\mu(g(x_1^n)) \geq \lambda_{\beta_c} = \max\{\mu(x_1), ..., \mu(x_n)\}$. Consequently, μ is a T-fuzzy hyperideal of R. ∎

5. The relationships among idealistic soft Krasner (m, n)-hyperrings, T-Fuzzy hyperideals and falling fuzzy hyperideals

In probability theory, a probability space is a triple $(\Omega, \mathbb{A}, P)$, where Ω is the set of all possible outcomes, $\mathbb{A}$ is a σ-algebra of subsets of Ω called events, and P is a probability measure on measurable space $(\Omega, \mathbb{A})$, i.e., P is a countable additive and positive function and $P(\Omega) = 1$. The following definitions are given in [36].

Let U be a universal set, and $\mathscr{P}(U)$ be the power set of U. Then for each $u \in U$, define $\dot{u} = \{A \mid u \in A \text{ and } A \subseteq U\}$. Also, for each $A \in \mathscr{P}(U)$, let $\dot{A} = \{\dot{u} \mid u \in A\}$. Then an ordered pair $(\mathscr{P}(U), \mathbb{B})$ is said to be a hyper-measure structure on U if $\mathbb{B}$ is a σ-field in $\mathscr{P}(U)$ and $\dot{U} \subseteq \mathbb{B}$.

Given a probability space $(\Omega, \mathbb{A}, P)$ and a hyper-measurable structure $(\mathscr{P}(U), \mathbb{B})$ on U, a random set on U is defined to be a mapping $\xi : \Omega \to \mathscr{P}(U)$ that is $\mathbb{A} - \mathbb{B}$ measurable, i.e., $\forall C \in \mathbb{B}$, $\xi^{-1}(C) = \{\omega \mid \omega \in \Omega \text{ and } \xi(\omega) \in C\} \in \mathbb{A}$.

Definition 5.1. Let $(\Omega, \mathbb{A}, P)$ be a probability space, and $\xi : \Omega \to \mathscr{P}(R)$ be a random set, where $\mathscr{P}(R)$ denotes the power set of R. If for any $\omega \in \Omega$, $\xi(\omega)$ is a hyperideal of R, then the falling shadow S of the random set ξ, i.e., $S(x) = P(\{\omega \mid x \in \xi(\omega)\})$ for all $x \in R$, is called a falling fuzzy hyperideal of R.

Theorem 5.2. *Let μ be a fuzzy set of R, and (F, A) be a $\in$-soft set over R with A=[0,1]. If (F, A) is an idealistic soft Krasner (m, n)-hyperring over R, then μ is a falling fuzzy hyperideal of R.*

Proof. Consider the probability space $(\Omega, \mathbb{A}, P) = ([0, 1], \sigma, m)$, where σ is a Borel field on $[0, 1]$ and m is the usual Lebesgue measure. Define $\xi : [0, 1] \to \mathscr{P}(R)$ by $\xi(t) = F(t)$. Since $\xi^{-1}(\dot{x}) = [0, \mu(x)] \subseteq [0, 1]$, ξ is a random set on R.

Furthermore, we obtain that the falling shadow S of ξ is μ, i.e., $S(x) = m(t \mid x \in \xi(t)) = m([0, \mu(x)]) = \mu(x)$. Since (F, A) is an idealistic soft Krasner (m, n)-hyperring over R, $F(t)$ is a hyperideal of R, for $t \in \mathrm{Supp}(F, A)$. Therefore, μ is a falling fuzzy hyperideal of R. ∎

The following example demonstrated that the converse Theorem 5.2 is not true.

Example 5.3. Consider the Krasner $(2, n)$-hyperring R described in Example 3.2. Let $(\Omega, \mathbb{A}, P) = ([0,1], \sigma, m)$ and $\xi : [0,1] \to \mathscr{P}(R)$ be defined by

$$\xi(t) = \begin{cases} \{0,1\} & \text{if } t \in [0, 0.45), \\ \{0,2\} & \text{if } t \in [0.45, 0.75), \\ \{0,1,2,3\} & \text{if } t \in [0.75, 1). \end{cases}$$

We can see that $\xi(t)$ is a hyperideal of R for all $t \in [0,1]$, then $S(x) = m(\{t \mid x \in \xi(t)\})$ is a falling fuzzy hyperideal of R, which is denoted by $S(0) = 1$, $S(1) = 0.7$, $S(2) = 0.55$ and $S(3) = 0.25$. Let (F, A) be a $\in$-soft set over R with A=[0,1]. Then $F : A \to \mathscr{P}(R)$ is a set-valued function described by

$$F(t) = \begin{cases} \{0,1,2,3\} & \text{if } t \in [0, 0.25], \\ \{0,1,2\} & \text{if } t \in (0.25, 0.55], \\ \{0,1\} & \text{if } t \in (0.55, 0.7], \\ \{0\} & \text{if } t \in (0.7, 1], \end{cases}$$

Since $\{0,1,2\}$ is not a hyperideal of R, (F, A) is not an idealistic soft Krasner $(2, n)$-hyperring over R.

Theorem 5.4. *If S is a falling fuzzy hyperideal of R, then S is a T_{Luk}-fuzzy hyperideal of R.*

Proof. For all $x_1^m \in \xi(\omega)$, we have $f(x_1^m) \subseteq \xi(\omega)$, since $\xi(\omega)$ is a hyperideal of R for any $\omega \in \Omega$. So for every $z \in f(x_1^m)$, we have

$$\{\omega \mid z \in \xi(\omega)\} \supseteq \bigcap_{i=1}^{m} \{\omega \mid x_i \in \xi(\omega)\}.$$

It follows that

$$\begin{aligned} S(z) = P(\{\omega \mid z \in \xi(\omega)\}) &\geq P(\bigcap_{i=1}^{m} \{\omega \mid x_i \in \xi(\omega)\}) \\ &= P(\bigcap_{i=1}^{m-1} \{\omega \mid x_i \in \xi(\omega)\}) + P(\{\omega \mid x_m \in \xi(\omega)\}) \\ &\quad - P(\{\omega \mid x_1^{m-1} \in \xi(\omega) \text{ or } x_m \in \xi(\omega)\}) \\ &\geq P(\bigcap_{i=1}^{m-1} \{\omega \mid x_i \in \xi(\omega)\}) + S(x_m) - 1 \\ &\vdots \end{aligned}$$

$$
\begin{aligned}
&\vdots \\
&\geq P(\{\omega \mid x_1 \in \xi(\omega)\} \cap \{\omega \mid x_2 \in \xi(\omega)\}) + \sum_{i=3}^{m} S(x_i) - (m-2) \\
&= P(\{\omega \mid x_1 \in \xi(\omega)\}) + P(\{\omega \mid x_2 \in \xi(\omega)\}) \\
&\quad -P(\{\omega \mid x_1 \in \xi(\omega) \text{ or } x_2 \in \xi(\omega)\}) + \sum_{i=3}^{m} S(x_i) - (m-2) \\
&\geq \sum_{i=1}^{m} S(x_i) - (m-1).
\end{aligned}
$$

Hence, we have

$$
\inf_{z \in f(x_1^m)} \{S(z)\} \geq \max\{\sum_{i=1}^{m} S(x_i) - (m-1), 0\} = T_{Łuk}(S(x_1), ..., S(x_m)).
$$

For any $x \in \xi(\omega)$, we have $-x \in \xi(\omega)$. Hence $\{\omega \mid -x \in \xi(\omega)\} \supseteq \{\omega \mid x \in \xi(\omega)\}$. Furthermore, we have $P\{\omega \mid -x \in \xi(\omega)\} \geq P\{\omega \mid x \in \xi(\omega)\}$, i.e., $S(-x) \geq S(x)$.

Since $\xi(\omega)$ is a hyperideal of R for any $\omega \in \Omega$, it follows that

$$
\{\omega \mid g(x_1^n) \in \xi(\omega)\} \supseteq \bigcup_{i=1}^{n} \{\omega \mid x_i \in \xi(\omega)\}.
$$

So we have

$$
\begin{aligned}
S(g(x_1^n)) &= P(\omega \mid g(x_1^n) \in \xi(\omega)) \geq P(\bigcup_{i=1}^{n} \{\omega \mid x_i \in \xi(\omega)\}) \\
&= P(\bigcup_{i=1}^{n-1} \{\omega \mid x_i \in \xi(\omega)\}) + P(\{\omega \mid x_n \in \xi(\omega)\}) \\
&-P(\bigcup_{i=1}^{n-1} \{\omega \mid x_i \in \xi(\omega)\} \ \cap \{\omega \mid x_m \in \xi(\omega)\}) \\
&\geq \max\{P(\bigcup_{i=1}^{n-1} \{\omega \mid x_i \in \xi(\omega)\}), S(x_n)\} \\
&\vdots \\
&\geq \max\{P(\{\omega \mid x_1 \in \xi(\omega)\} \cup \{\omega \mid x_2 \in \xi(\omega)\}), S(x_3), ..., S(x_n)\} \\
&= \max\{P(\{\omega \mid x_1 \in \xi(\omega)\}) + P(\{\omega \mid x_2 \in \xi(\omega)\}) - P(\{\omega \mid x_1 \in \xi(\omega) \\
&\quad \text{and } x_2 \in \xi(\omega)\}), S(x_3), ..., S(x_n)\} \\
&\geq \max\{S(x_1), ..., S(x_n)\}..
\end{aligned}
$$

Therefore, S is a $T_{Łuk}$-fuzzy hyperideal of R. ∎

Let R be a Krasner (m, n)-ary hyperring, and Ω be a non-empty set. Define

$$
\mathscr{P}(R)^{\Omega} = \{\eta \mid \eta : \Omega \to \mathscr{P}^*(R) \text{ is a mapping}\}
$$

Let $\eta_i \in \mathscr{P}(R)^\Omega$, $i = 1, ..., m$. For any $\omega \in \Omega$, we put $\eta_i(\omega) = \{y^\omega_{i1}, ..., y^\omega_{i|\eta_i(\omega)|}\}$, $i = 1, ..., m$. Suppose

$$\begin{aligned}\overline{f}(\eta_1, ..., \eta_m) &= \{\zeta_{(t_1,...,t_m)} \mid \zeta_{(t_1,...,t_m)}(\omega) = f(y^\omega_{1t_1}, ..., y^\omega_{mt_m}), t_i \in \{1, ..., |\eta_i(\omega)|\}, \\ &\qquad i = 1, ..., m\}, \\ \overline{g}(\eta_1, ..., \eta_n)(\omega) &= \{g(y^\omega_{1t_1}, ..., y^\omega_{nt_n}) \mid t_i \in \{1, ..., |\eta_i(\omega)|\}, i = 1, ..., n\}.\end{aligned}$$

Then $(\mathscr{P}(R)^\Omega, \overline{f}, \overline{g})$ is a Krasner (m, n)-ary hyperring.

Theorem 5.5. Let $(\Omega, \mathbb{A}, P)$ be a probability space, I be a hyperideal of a Krasner (m, n)-ary hyperring R and B be a hyperideal of $\mathscr{P}(R)^\Omega$ such that $S_\eta = \{\omega \in \Omega \mid \eta(\omega) \subseteq I\} \in \mathbb{A}$ for all $\eta \in B$. Define $\mu : B \to [0, 1]$ by $\mu(\eta) = P(S_\eta)$ for all $\eta \in B$. Then μ is a $T_{\text{Ł}uk}$-fuzzy hyperideal of B.

Proof. Let $\eta_1, ..., \eta_m \in B$. For every $\omega \in \bigcap\limits_{i=1}^{m} S_{\eta_i}$, we have $\eta_i(\omega) \subseteq I$, where $i = 1, ..., m$, that is, $y^\omega_{ij} \in I$, where $i = 1, ..., m$ and $j = 1, ..., |\eta_i(\omega)|$. Thus, for all $\zeta \in \overline{f}(\eta_1, ..., \eta_m)$, we have $\zeta(\omega) \subseteq I$. So $\omega \in S_\zeta$. Consequently, $S_\zeta \supseteq \bigcap\limits_{i=1}^{m} S_{\eta_i}$. Furthermore,

$$\mu(\zeta) = P(S_\zeta) \geq P(\bigcap_{i=1}^{m} S_{\eta_i}) \geq \max\{\sum_{i=1}^{m} P(S_{\eta_i}) - (m-1), 0\} = T_{\text{Ł}uk}(\mu(\eta_1), ..., \mu(\eta_m)).$$

Hence, we have $\inf\limits_{\zeta \in \overline{f}(\eta_1,...,\eta_m)} \{\mu(\zeta)\} \geq T_{\text{Ł}uk}(\mu(\eta_1), ..., \mu(\eta_m))$.

Let $\eta \in B$. For every $\omega \in S_\eta$, $\eta(\omega) = \{y_1, ..., y_{|\eta(\omega)|}\} \subseteq I$. Hence, $-\eta(\omega) = \{-y_1, ..., -y_{|\eta(\omega)|}\} \subseteq I$, and so $\omega \in S_{-\eta}$. Therefore, we have $S_{-\eta} \supseteq S_\eta$. It follows that $\mu(-\eta) = P(S_{-\eta}) \geq P(S_\eta) = \mu(\eta)$.

Let $\eta_1, ..., \eta_n \in B$. If $\omega \in \bigcup\limits_{i=1}^{n} S_{\eta_i}$, then there exists $i \in \{1, ..., n\}$ such that $\eta_i(\omega) = \{y^\omega_{i1}, ..., y^\omega_{i|\eta_i(\omega)|}\} \subseteq I$. Hence, $\overline{g}(\eta_1, ..., \eta_n)(\omega) \subseteq I$ since I is a hyperideal of R. So we have $\omega \in S_{\overline{g}(\eta_1,...,\eta_n)}$. It follows that $S_{\overline{g}(\eta_1,...,\eta_n)} \supseteq \bigcup\limits_{i=1}^{n} S_{\eta_i}$. Therefore,

$$\begin{aligned}\mu(\overline{g}(\eta_1, ..., \eta_n)) &= P(S_{\overline{g}(\eta_1,...,\eta_n)}) \geq P(\bigcup_{i=1}^{n} S_{\eta_i}) \geq \max\{P(S_{\eta_1}), ..., P(S_{\eta_n})\} \\ &= \max\{\mu(\eta_1), ..., \mu(\eta_n)\}.\end{aligned}$$

Therefore, μ is a $T_{\text{Ł}uk}$-fuzzy hyperideal of R. ∎

Based on the theory of falling shadows, Tan et al. [34] establish a theoretical approach to define a fuzzy inference relation. Let A and B be fuzzy sets in the universes U and V, respectively, ξ and η be cut-clouds of A and B, respectively. Then the fuzzy inference relation $I_{A\to B}$ of the implication $A \to B$ is defined to be

$$I_{A\to B}(u, v) = P((\lambda, \mu)|(u, v) \in I_{A_\lambda \to B_\mu}) = P((\lambda, \mu)|(u, v) \in (A_\lambda \times B_\mu) \cup (A^c_\lambda \times V)),$$

where P is a joint probability on $[0,1]^2$. So different probability distribution P will generate different formula for the fuzzy inference relation. The following three basic cases are considered.

(1) If the whole probability P of (λ,μ) on $[0,1]^2$ is concentrated and uniformly distributed on the main diagonal $\{(\lambda,\lambda)|\lambda\in[0,1]\}$ of the unit square $[0,1]^2$, then P is the diagonal distribution and $I_{A\to B}(u,v)=\min(1-A(u)+B(v),1)$.

(2) If the whole probability P of (λ,μ) on $[0,1]^2$ is concentrated and uniformly distributed on the anti-diagonal $\{(\lambda,1-\lambda)|\lambda\in[0,1]\}$ of the unit square $[0,1]^2$, then P is the anti-diagonal distribution and $I_{A\to B}(u,v)=\max(1-A(u),B(v))$.

(3) If the whole probability P of (λ,μ) on $[0,1]^2$ is uniformly distributed on the unit square $[0,1]^2$, then P is the independent distribution and $I_{A\to B}(u,v)=1-A(u)+A(u)B(v)$.

We call the three fuzzy inference relations falling implication operators on $[0,1]$.

Definition 5.6. Let μ be a fuzzy set of R, I_{FS} be a falling implication operator over $[0,1]$ and $\lambda\in(0,1]$. Then μ is called an I_{FS}-fuzzy hyperideal of R If the following conditions are satisfied:

(1) $I_{FS}(\min\{\mu(x_1),...,\mu(x_m)\},\inf\limits_{z\in f(x_1^m)}\{\mu(z)\})\geq\lambda$ for all $x_1^m\in R$;

(2) $I_{FS}(\mu(x),\mu(-x))\geq\lambda$ for all $x\in R$;

(3) $I_{FS}(\max\{\mu(x_1),...,\mu(x_n)\},\mu(g(x_1^n)))\geq\lambda$ for all $x_1^n\in R$.

Clearly, if $\lambda=1$ and P is the diagonal distribution, then Definition 5.6 is equivalent to Definition 4.1.

Theorem 5.7. *Let μ be a fuzzy set of R and $\lambda=0.5$, then*

(1) *if P is the diagonal distribution, then μ is an I_{FS}-fuzzy hyperideal of R, if and only if*

  (a) $\min\{\mu(x_1),...,\mu(x_m)\}\leq\inf\limits_{z\in f(x_1^m)}\{\mu(z)\}$, *or*
  $0<\min\{\mu(x_1),...,\mu(x_m)\}-\inf\limits_{z\in f(x_1^m)}\{\mu(z)\}\leq 0.5$ *for all* $x_1^m\in R$,

  (b) $\mu(x)\leq\mu(-x)$ *or* $0\leq\mu(x)-\mu(-x)\leq 0.5$ *for all* $x\in R$,

  (c) $\max\{\mu(x_1),...,\mu(x_n)\}\leq\mu(g(x_1^n)$, *or*
  $0\leq\max\{\mu(x_1),...,\mu(x_n)\}-\mu(g(x_1^n))\leq 0.5$ *for all* $x_1^n\in R$;

(2) *if P is the anti-diagonal distribution, then μ is an I_{FS}-fuzzy hyperideal of R, if and only if*

(a) $\min\{\mu(x_1), ..., \mu(x_m)\} \leq \inf_{z \in f(x_1^m)} \{\max\{\mu(z), 0.5\}\}$, *or*
$\min\{\mu(x_1), ..., \mu(x_m), 0.5\} \leq \inf_{z \in f(x_1^m)} \{\mu(z)\}$ *for all* $x_1^m \in R$,

(b) $\mu(x) \leq \max\{\mu(-x), 0.5\}$ *or* $\min\{\mu(x), 0.5\} \leq \mu(-x)$ *for all* $x \in R$,

(c) $\max\{\mu(x_1), ..., \mu(x_n)\} \leq \max\{\mu(g(x_1^n)), 0.5\}$, *or*
$\max\{\min\{\mu(x_1), 0.5\}, ..., \min\{\mu(x_n), 0.5\}\} \leq \mu(g(x_1^n)$ *for all* $x_1^n \in R$;

(3) *if P is the independent distribution, then μ is an I_{FS}-fuzzy hyperideal of R, if and only if*

(a) $\min\{\mu(x_1), ..., \mu(x_m)\}(1 - \inf_{z \in f(x_1^m)} \{\mu(z)\}) \leq 0.5$ *for all* $x_1^m \in R$,

(b) $\mu(x)(1 - \mu(-x)) \leq 0.5$ *for all* $x \in R$,

(c) $\max\{\mu(x_1), ..., \mu(x_n)\}(1 - \mu(g(x_1^n))) \leq 0.5$ *for all* $x_1^n \in R$.

Proof. We only prove (1), and the proofs of (2) and (3) are similar. P is the diagonal distribution, and μ is an I_{FS}-fuzzy hyperideal of R, then we have

(a) $\min(1 - \min\{\mu(x_1), ..., \mu(x_m)\} + \inf_{z \in f(x_1^m)} \{\mu(z)\}, 1) \geq 0.5$ for all $x_1^m \in R$;

(b) $\min(1 - \mu(x) + \mu(-x), 1) \geq 0.5$ for all $x \in R$;

(c) $\min(1 - \max\{\mu(x_1), ..., \mu(x_n)\} + \mu(g(x_1^n)), 1) \geq 0.5$ for all $x_1^n \in R$.

It follows that

(a) $\min(1 - \min\{\mu(x_1), ..., \mu(x_m)\} + \inf_{z \in f(x_1^m)} \{\mu(z)\}, 1) \geq 0.5$
$\Leftrightarrow 1 - \min\{\mu(x_1), ..., \mu(x_m)\} + \inf_{z \in f(x_1^m)} \{\mu(z)\} \geq 1$, or
$1 \geq 1 - \min\{\mu(x_1), ..., \mu(x_m)\} + \inf_{z \in f(x_1^m)} \{\mu(z)\} \geq 0.5$
$\Leftrightarrow \min\{\mu(x_1), ..., \mu(x_m)\} \leq \inf_{z \in f(x_1^m)} \{\mu(z)\}$, or
$0 < \min\{\mu(x_1), ..., \mu(x_m)\} - \inf_{z \in f(x_1^m)} \{\mu(z)\} \leq 0.5$;

(b) $\min(1 - \mu(x) + \mu(-x), 1) \geq 0.5 \Leftrightarrow 1 - \mu(x) + \mu(-x) \geq 1$, or
$1 \geq 1 - \mu(x) + \mu(-x) \geq 0.5 \Leftrightarrow \mu(x) \leq \mu(-x)$, or
$0 \leq \mu(x) - \mu(-x) \leq 0.5$;

(c) $\min(1 - \max\{\mu(x_1), ..., \mu(x_n)\} + \mu(g(x_1^n)), 1) \geq 0.5$
$\Leftrightarrow 1 - \max\{\mu(x_1), ..., \mu(x_n)\} + \mu(g(x_1^n)) \geq 1$, or
$1 \geq 1 - \max\{\mu(x_1), ..., \mu(x_n)\} + \mu(g(x_1^n)) \geq 0.5$
$\Leftrightarrow \max\{\mu(x_1), ..., \mu(x_n)\} \leq \mu(g(x_1^n)$, or
$0 \leq \max\{\mu(x_1), ..., \mu(x_n)\} - \mu(g(x_1^n)) \leq 0.5$. ∎

6. Conclusions

In this paper, soft Krasner (m, n)-hyperrings and idealistic soft Krasner (m, n)-hyperrings are introduced based on soft set theory, and three isomorphism theorems of soft Krasner (m, n)-hyperrings are derived. Furthermore, we consider T-fuzzy hyperideals of Krasner (m, n)-hyperrings by using triangular norms, and discuss the relationships among idealistic soft Krasner (m, n)-hyperrings, fuzzy hyperideals and T-fuzzy hyperideals. Also, we define falling fuzzy hyperideals of Krasner (m, n)-hyperrings by using the theory of falling shadows, and give the relationships among idealistic soft Krasner (m, n)-hyperrings, T-fuzzy hyperideals and falling fuzzy hyperideals. In the following work, we will consider the connection of fuzzy soft set theory and some algebraic hyperstructures.

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