FUZZY PARAMETERIZED FUZZY SOFT RINGS AND APPLICATIONS

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Abstract. The concepts of FP-fuzzy soft rings, FP-equivalent fuzzy soft rings and FP-increasing(decreasing) fuzzy soft rings are introduced. Then some properties of them are given. Finally, aggregate fuzzy subrings are proposed by aggregate fuzzy sets of FP-fuzzy soft rings.

Keywords: FP-fuzzy soft rings; FP-equivalent fuzzy soft rings; FP-increasing (decreasing) fuzzy soft rings; FP-fuzzy soft homomorphism; aggregate fuzzy subrings.

2010 Mathematics Subject Classification: 16Y60; 13E05; 03G25.

1. Introduction

In dealing with the complicated problems in economics, engineering and environmental sciences, we are usually unable to apply the classical methods because there are various uncertainties in these problems. Some kinds of theories were developed like theory of fuzzy sets, soft sets, i.e., which can be used as the fundamental tools for dealing with uncertainties.

The concept of fuzzy sets and fuzzy set operations, introduced by L.A. Zadeh [18], have been extensively applied to many scientific fields. In 1971, A. Rosenfeld [17] applied the concept to the theory of groupoids and groups. In 1982, W. Liu [8] defined and studied fuzzy subrings as well as fuzzy ideals. Since then many papers concerning various fuzzy algebraic structures have appeared in the literature.

The concept of soft sets was introduced by D. Molodtsov in 1999 [15], which was another mathematical tool for dealing with uncertainties. At present, the algebraic structure of set theories dealing with uncertainties has been studied by

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many authors. H. Aktaş et al. [1] applied the notion of soft sets to the theory of groups. Y.B. Jun [6] introduced the notions of soft BCK/BCI-algebras, and then investigated their basic properties [7]. We also noticed that F. Feng et al. [5] have already investigated the structure of soft semirings. In [9], we have proposed the definition of soft rings and established three isomorphism theorems. Furthermore, we gave three fuzzy isomorphism theorems of soft rings in [10].

In 2001, P.K. Maji et al. [13] presented the definition of fuzzy soft set, and Roy et al. presented some applications of this notion to decision-making problems in [14]. We notice that E. İnan et al. [4] have already introduced the definition of fuzzy soft rings and studied some of their basic properties.

Furthermore, N. Çağman introduced fuzzy parameterized soft sets [2] and fuzzy parameterized fuzzy soft sets [3], in short written FP-soft sets and FP-fuzzy soft sets, respectively, whose parameters sets are fuzzy sets and have improved several results. In [3], the authors also defined their operation and soft aggregation operator to form FP-fuzzy soft decision making method that allows constructing more efficient decision processes. FP-soft sets and FP-fuzzy soft sets have already been studied by some authors. We have studied FP-soft rings on FP-soft set theory in [11].

In this paper, we study FP-fuzzy soft rings on FP-fuzzy soft set theory. We first introduce FP-fuzzy soft rings generated by FP-fuzzy soft sets and some properties of FP-fuzzy soft rings will be given. Then FP-equivalent soft rings and FP-accelerating(decelerating) fuzzy soft rings will be studied. Moreover, the notions of FP-fuzzy homomorphisms of FP-fuzzy soft rings are proposed and some examples are given. Finally, aggregate fuzzy subrings will be proposed by aggregate operator and an example will be given to show that the methods can be successfully applied to many problems that contain uncertainties.

2. Preliminaries

Definition 2.1 [16]

(i) A fuzzy set μ in a ring R is said to be a fuzzy subring of R if the following conditions hold for all $x, y \in R$:

(1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$, and

(2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$.

(ii) A fuzzy set μ in a ring R is said to be a fuzzy left (right) ideal of R if the following conditions hold for all $x, y \in R:$ (1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$, and (3) $\mu(xy) \geq \mu(y)$ $(\mu(xy) \geq \mu(x))$.

(iii) A fuzzy set μ is said to be a fuzzy ideal of R if it is both a fuzzy left ideal of R and a fuzzy right ideal of R.

Definition 2.2 [12] Let $f : X \to Y$ be a mapping of sets, μ a fuzzy set of X and ν a fuzzy set of Y. Then the image $f(\mu)$ of μ and preimage $f^{(-1)}(\nu)$ of ν are both fuzzy sets defined respectively as follows:

$$
\begin{aligned}
f(\mu)(y) &= \begin{cases} \sup\limits_{x \in f^{(-1)}(y)} \mu(x) & \text{if } f^{(-1)}(y) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases} \\
f^{(-1)}(\nu)(x) &= \nu(f(x)), x \in X.
\end{aligned}
$$

Definition 2.3 [3] Let U be an initial universe, E be the set of all parameters and X be a fuzzy set over E with the membership function $\mu_X : E \longrightarrow [0,1]$ and $\gamma_X(x)$ be a fuzzy set over U for all $x \in E$, $F(U)$ be the set of all fuzzy set of U. Then an fuzzy parameterized fuzzy soft set Γ_X on U is defined by a function $\gamma_X(x)$ representing a mapping

$$\gamma_X : E \longrightarrow F(U)$$

such that $\gamma_X(x) = \emptyset$ if $\mu_X(x) = 0$. Here, γ_X is called the fuzzy approximate function of the fuzzy parameterized fuzzy soft set Γ_X, and the value $\gamma_X(x)$ is a fuzzy set called $x-$element of the fuzzy parameterized fuzzy soft set for all $x \in E$. Thus a fuzzy parameterized fuzzy soft set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(U), \mu_X(x) \in [0,1]\}.$$

A fuzzy parameterized fuzzy soft set is briefly said to be an FP-fuzzy soft set. The set of all FP-fuzzy soft sets is denoted by $FPFS(U)$.

Definition 2.4 [3] Let $\Gamma_X \in FPFS(U)$.

(i) If $\gamma_X(x) = \emptyset$ for all $x \in E$, then Γ_X is called an X-empty FP-fuzzy soft set, denoted by $\Gamma_{\emptyset_X}$.

(ii) If $X = \emptyset$, then the Γ_X is called an empty FP-fuzzy soft set, denoted by $\Gamma_\emptyset$.

(iii) If $\mu_X(x) = 1$ and $\gamma_X(x) = U$ for all $x \in E$, then Γ_X is called an X-universal FP-fuzzy soft set, denoted by $(\Gamma_U)_X$.

(iv) If $X = E$, then the X-universal FP-fuzzy soft set is called an universal FP-fuzzy set, denoted by Γ_E.

Definition 2.5 [3] Let $\Gamma_X, \Gamma_Y \in FPFS(U)$. Then

(i) Γ_X is an FP-fuzzy soft subset of Γ_Y, denoted by $\Gamma_X \widetilde{\subseteq} \Gamma_Y$, if $\mu_X(x) \leq \mu_Y(x)$ and $\gamma_X(x) \subseteq \gamma_Y(x)$ for all $x \in E$.

(ii) Γ_X and Γ_Y are FP-equal, denoted by $\Gamma_X = \Gamma_Y$, if $\mu_X(x) = \mu_Y(x)$ and $\gamma_X(x) = \gamma_Y(x)$ for all $x \in E$.

Definition 2.6 [3] Let $\Gamma_X \in FPFS(U)$. Then the complement of Γ_X, denoted by $\Gamma_X^{\bar{c}}$, is an FP-fuzzy soft set defined by

$$\mu_X^{\bar{c}}(x) = 1 - \mu(x) \text{ and } \gamma_X^{\bar{c}}(x) = U \setminus \gamma_X(x).$$

Definition 2.7 [3] Let $\Gamma_X, \Gamma_Y \in FPFS(U)$.

(i) The union of Γ_X and Γ_Y, denoted by $\Gamma_X \widetilde{\cup} \Gamma_Y$, is defined by $\mu_{X\widetilde{\cup}Y}(x) = \max\{\mu_X(x), \mu_Y(x)\}$ and $\gamma_{X\widetilde{\cup}Y}(x) = \gamma_X(x) \cup \gamma_Y(x)$ for all $x \in E$.

(ii) The intersection of Γ_X and Γ_Y, denoted by $\Gamma_X \widetilde{\cap} \Gamma_Y$, is defined by $\mu_{X\widetilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\}$ and $\gamma_{X\widetilde{\cap}Y}(x) = \gamma_X(x) \cap \gamma_Y(x)$ for all $x \in E$.

3. Fuzzy parameterized fuzzy soft rings

Definition 3.1 Let R be a ring, E be a set of parameters and X be a fuzzy set over E, $\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\} \in FPFS(R)$. Then Γ_X is said to be an FP-fuzzy soft ring over R if, for any $x \in E$, $\gamma_X(x)$ is a fuzzy subring of R.

Example 3.2 Let $R = Z_4 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ be a ring and $E = \{a, b\}$ be a set of parameters. If $X = \{0.2/a, 0.4/b\}$, $\gamma_X(a) = \{\overline{0}/0.5, \overline{1}/0.3, \overline{2}/0.4, \overline{3}/0.3\}$, $\gamma_X(b) = \{\overline{0}/0.4, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$, then Γ_X is an FP-fuzzy soft ring over R.

Theorem 3.3 *Let R be a ring, E be a set of parameters. Then*

(1) $\Gamma_{\emptyset_X}$ *and* $\Gamma_\emptyset$ *are FP-fuzzy soft rings.*

(2) $(\Gamma_R)_X$ *and* $(\Gamma_R)_X^{\bar{c}}$ *are FP-fuzzy soft rings.*

(3) $\Gamma_\emptyset^{\bar{c}}$ *and* $\Gamma_E^{\bar{c}}$ *are FP-fuzzy soft rings.*

Proof. By Definitions 2.4 and 3.1, the proofs of (1) and (2) are straightforward. Since $\Gamma_\emptyset^{\bar{c}} = (\Gamma_R)_E$ and $\Gamma_E^{\bar{c}} = \Gamma_\emptyset$, then (3) is hold. ∎

Theorem 3.4 *Let R be a ring, E be a set of parameters and Γ_X and Γ_Y be FP-fuzzy soft rings over R. Then their intersection $\Gamma_X \widetilde{\cap} \Gamma_Y$ is still an FP-fuzzy soft ring over R.*

Proof. We can write $\Gamma_X \widetilde{\cap} \Gamma_Y = \Gamma_{X\widetilde{\cap}Y}$. For all $x \in E$, $\mu_{X\widetilde{\cap}Y}(x) = \min\{\mu_X(x), \mu_Y(x)\}$, $\gamma_X(x)$ and $\gamma_Y(x)$ are fuzzy subrings of R, then $\gamma_{X\widetilde{\cap}Y}(x) = \gamma_X(x) \cap \gamma_Y(x)$ is a fuzzy subring of R. Therefore, $\Gamma_X \widetilde{\bigcap} \Gamma_Y$ is an FP-fuzzy soft ring over R. ∎

Theorem 3.5 *Let R be a ring, E be a set of parameters and Γ_X and Γ_Y be FP-fuzzy soft rings over R with $X \cap Y = \emptyset$. Then their union $\Gamma_X \widetilde{\cup} \Gamma_Y$ is still an FP-fuzzy soft ring over R.*

Proof. We can write $\Gamma_X \widetilde{\cup} \Gamma_Y = \Gamma_{X\widetilde{\cup}Y}$. For all $x \in E$, $\mu_{X\widetilde{\cup}Y}(x) = \max\{\mu_X(x), \mu_Y(x)\}$, then $\mu_{X\widetilde{\cup}Y}(x) = \mu_X(x)$ or $\mu_{X\widetilde{\cup}Y}(x) = \mu_Y(x)$ since $X \cap Y = \emptyset$. Therefore, $\gamma_{X\widetilde{\cup}Y}(x) = \gamma_X(x)$ or $\gamma_{X\widetilde{\cup}Y}(x) = \gamma_Y(x)$, so $\gamma_{X\widetilde{\cup}Y}(x)$ is a fuzzy subring of R, then $\Gamma_X \widetilde{\bigcup} \Gamma_Y$ is an FP-fuzzy soft ring over R. ∎

Definition 3.6 Let Γ_X, Γ_Y be FP-fuzzy soft rings over R. Then Γ_X is said to be an FP-fuzzy soft subring of Γ_Y, if $\mu_X(x) \leq \mu_Y(x)$ and $\gamma_X(x)$ is a fuzzy subset of $\gamma_Y(x)$ for all $x \in E$.

Example 3.7 Let $R = Z_4 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ be a ring and $E = \{a, b\}$ be a set of parameters. If $X = \{0.2/a, 0/b\}$, $\gamma_X(a) = \{\overline{0}/0.5, \overline{1}/0.3, \overline{2}/0.4, \overline{3}/0.3\}$, $\gamma_X(b) = \emptyset$, and $Y=\{0.4/a, 0.3/b\}$, $\gamma_Y(a)=\{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$, $\gamma_Y(b) = \{\overline{0}/0.4, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$, then Γ_X and Γ_Y are FP-fuzzy soft rings over R, and Γ_X is an FP-fuzzy soft subring of Γ_Y.

Theorem 3.8 *Let R be a ring, E be a set of parameters, Γ_X and Γ_Y are FP-fuzzy soft subrings of Γ_Z.*

(1) $\Gamma_X \widetilde{\bigcap} \Gamma_Y$ *is an FP-fuzzy soft subring of Γ_Z.*

(2) *If $X \cap Y = \emptyset$, then $\Gamma_X \widetilde{\bigcup} \Gamma_Y$ is an FP-fuzzy soft subring of Γ_Z.*

Proof. The proofs are similar to the proofs of Theorems 3.4 and 3.5. ∎

Definition 3.9 Let $\Gamma_X=\{(\mu_X(x)/x, \gamma_X(x)) : x \in A, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ and $\Gamma_Y = \{(\mu_Y(y)/y, \gamma_Y(y) : y \in B, f_Y(y) \in F(K), \mu_Y(y) \in [0,1]\}$ be FP-fuzzy soft rings over rings R and K, respectively. If $f : R \to K$ and $g : A \to B$ are two functions, then (f, g) is called an FP-fuzzy soft homomorphism such that (f, g) is an FP-fuzzy soft homomorphism from Γ_X to Γ_Y. The latter is written by $\Gamma_X \sim \Gamma_Y$ if the following conditions are satisfied:

(1) f is an epimorphism from R to K,

(2) g is a surjective mapping, and

(3) $f(\gamma_X(x)) = \gamma_Y(g(x))$ and $\mu_X(x) = \mu_Y(g(x))$ for all $x \in A$.

In the above definition, if f is an isomorphism from R to K and g is a bijective mapping, then (f, g) is called an FP-fuzzy soft isomorphism so that (f, g) is an FP-fuzzy soft isomorphism from Γ_X to Γ_Y, denoted by $\Gamma_X \simeq \Gamma_Y$.

Example 3.10 Let $R=(Z, +, \times)$ and $K=(4Z, +, \times)$, $A=\{1, 3\}$ and $B=\{2, 6\}$. Define a homomorphism f from R onto K by $f(r) = 4r$ for $r \in R$, and a mapping g from A onto B by $g(x) = 2x$, for $x \in A$.

Let X be a fuzzy set over A defined by $\mu_X = \{1/0.5, 3/0.8\}$,

Let Y be a fuzzy set over B defined by $\mu_Y = \{2/0.5, 6/0.8\}$.

Let $\gamma_X : A \to F(R)$ defined by

$$(\gamma_X(1))(r) = \begin{cases} 0.1, & r = 2k+1, k \in Z, \\ 0.3, & r = 2k, k \in Z. \end{cases}$$

$$(\gamma_X(3))(r) = \begin{cases} 0.2, & r = 2k+1, k \in Z, \\ 0.4, & r = 2k, k \in Z. \end{cases}$$

Let $\gamma_Y : B \to F(K)$ defined by

$$(\gamma_Y(2))(r) = \begin{cases} 0.1, & r = 8k+4, k \in Z, \\ 0.3, & r = 8k, k \in Z. \end{cases}$$

$$(\gamma_Y(6))(r) = \begin{cases} 0.2, & r = 8k+4, k \in Z, \\ 0.4, & r = 8k, k \in Z. \end{cases}$$

It is clear that Γ_X and Γ_Y are FP-fuzzy soft rings over R and K, respectively. We can immediately see that f is an isomorphism from R to K and g is a bijective mapping, $\mu_X(x) = \mu_Y(g(x))$ and we can deduce that $f(\gamma_X(x)) = \gamma_Y(g(x))$ for all $x \in A$. Hence (f, g) is an FP-fuzzy soft isomorphism from Γ_X to Γ_Y.

The following lemma is similar to fuzzy subgroups in [12], and we omit the proof.

Lemma 3.11 *If $f : R \to K$ is an epimorphism of rings and μ a fuzzy subring(ideal) of R, then $f(\mu)$ is a fuzzy subring(ideal) of K.*

Theorem 3.12 *Let $\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in A, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ be an FP-fuzzy soft ring over R and $\Gamma_Y = \{(\mu_Y(y)/y, \gamma_Y(y)) : y \in B, \gamma_Y(y) \in F(K)$, $\mu_Y(y) \in [0,1]\}$ be an FP-fuzzy soft set over ring K. If Γ_X is FP-fuzzy soft homomorphic to Γ_Y, then Γ_Y is an FP-fuzzy soft ring over K.*

Proof. Let (f, g) be an FP-fuzzy soft homomorphism from Γ_X to Γ_Y. Since Γ_X is an FP-fuzzy soft ring over R, $f(R) = K$ and $\gamma_X(x)$ is a fuzzy subring of R for all $x \in A$. Now, for all $y \in B$, there exists $x \in A$ such that $g(x) = y$. Hence, $\gamma_Y(y) = \gamma_Y(g(x)) = f(\gamma_X(x))$ is a fuzzy subring of the ring K and $\mu_Y(y) = \mu_Y(g(x)) = \mu_X(x)$, so Γ_Y must be an FP-fuzzy soft ring over K as well. ■

4. FP-equivalent fuzzy soft rings

Definition 4.1 Let $\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ be an FP-fuzzy soft ring over R. Then Γ_X is said to be FP-equivalent fuzzy soft ring over R if, for any $x, y \in E$, $\mu_X(x) = \mu_X(y)$, we have $\gamma_X(x) = \gamma_X(y)$.

Example 4.2 Let $R = Z_4$, $E = \{x_1, x_2, x_3, x_4\}$ be and X be a fuzzy set over E defined by $X = \{0.1/x_1, 0.5/x_2, 0.5/x_3, 0.3/x_4\}$, $\gamma_X(x_1) = \{\overline{0}/1, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$, $\gamma_X(x_2) = \{\overline{0}/0.8, \overline{1}/0.5, \overline{2}/0.6, \overline{3}/0.5\}$, $\gamma_X(x_3) = \{\overline{0}/0.8, \overline{1}/0.5, \overline{2}/0.6, \overline{3}/0.5\}$, $\gamma_X(x_4) = \{\overline{0}/0.7, \overline{1}/0.3, \overline{2}/0.5, \overline{3}/0.3\}$, It is clearly that Γ_X is an FP-equivalent fuzzy soft ring over R.

Theorem 4.3 *Let R be a ring, E be a set of parameters. Then*

(1) $\Gamma_{\emptyset_X}$ *and* $\Gamma_\emptyset$ *are* FP*-equivalent fuzzy soft rings.*

(2) $(\Gamma_R)_X$ *and* $(\Gamma_R)_X^{\tilde{c}}$ *are* FP*-equivalent fuzzy soft rings.*

(3) $\Gamma_\emptyset^{\tilde{c}}$ *and* $\Gamma_E^{\tilde{c}}$ *are* FP*-equivalent fuzzy soft rings.*

Proof. By Definitions 2.4 and 4.1, the proofs of (1) and (2) are straightforward. Since $\Gamma_\emptyset^{\tilde{c}} = (\Gamma_R)_E$ and $\Gamma_E^{\tilde{c}} = \Gamma_\emptyset$, then (3) is hold. ■

Notation 4.4 If $\Gamma_X = \{\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ and $\Gamma_Y = \{(\mu_Y(x)/x, \gamma_Y(x)) : x \in E, \gamma_Y(x) \in F(R), \mu_Y(x) \in [0,1]\}$ are FP-equivalent fuzzy soft rings over ring R, $\Gamma_X \widetilde{\cap} \Gamma_Y$ is not always an FP-equivalent fuzzy soft ring over R.

Example 4.5 Let $R = Z_4$, $E = \{x_1, x_2, x_3, x_4\}$. Let Γ_X be an FP-fuzzy soft set over R defined by
$X = \{0.1/x_1, 0.5/x_2, 0.5/x_3, 0.3/x_4\}$,
$\gamma_X(x_1) = \{\overline{0}/1, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$, $\gamma_X(x_2) = \{\overline{0}/0.8, \overline{1}/0.5, \overline{2}/0.6, \overline{3}/0.5\}$,
$\gamma_X(x_3) = \{\overline{0}/0.8, \overline{1}/0.5, \overline{2}/0.6, \overline{3}/0.5\}$, $\gamma_X(x_4) = \{\overline{0}/0.7, \overline{1}/0.3, \overline{2}/0.5, \overline{3}/0.3\}$.
And let Γ_Y be an FP-fuzzy soft set over R defined by
$Y = \{0.3/x_1, 0.3/x_2, 0.6/x_3, 0.1/x_4\}$,
$\gamma_Y(x_1) = \{\overline{0}/0.5, \overline{1}/0.3, \overline{2}/0.4, \overline{3}/0.3\}$, $\gamma_Y(x_2) = \{\overline{0}/0.5, \overline{1}/0.3, \overline{2}/0.4, \overline{3}/0.3\}$,
$\gamma_Y(x_3) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$, $\gamma_Y(x_4) = \{\overline{0}/0.8, \overline{1}/0.2, \overline{2}/0.7, \overline{3}/0.2\}$,
It is clearly that Γ_X and Γ_Y are FP-equivalent fuzzy soft rings over R.
We can see that
$X \cap Y = \{0.1/x_1, 0.3/x_2, 0.5/x_3, 0.1/x_4\}$, $\mu_{(X\cap Y)}(x_1) = \mu_{(X\cap Y)}(x_4)$, but
$\gamma_{(X\cap Y)}(x_1) = \{\overline{0}/0.5, \overline{1}/0.3, \overline{2}/0.4, \overline{3}/0.3\}$,
$\gamma_{(X\cap Y)}(x_4) = \{\overline{0}/0.7, \overline{1}/0.2, \overline{2}/0.5, \overline{3}/0.2\}$.
Then $\Gamma_X \widetilde{\cap} \Gamma_Y$ is not an FP-equivalent fuzzy soft ring over R.

Notation 4.6 If $\Gamma_X = \{\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ and $\Gamma_Y = \{(\mu_Y(x)/x, \gamma_Y(x)) : x \in E, \gamma_Y(x) \in F(R), \mu_Y(x) \in [0,1]\}$ are FP-equivalent fuzzy soft rings over ring R with $X \cap Y = \emptyset$, $\Gamma_X \widetilde{\cup} \Gamma_Y$ is not always an FP-equivalent fuzzy soft ring over R.

Example 4.7 Let $R = Z_4$, $E = \{x_1, x_2, x_3, x_4\}$. Let Γ_X be an FP-fuzzy soft set over R defined by
$X = \{0/x_1, 0.5/x_2, 0.5/x_3, 0/x_4\}$,
$\gamma_X(x_1) = \emptyset$, $\gamma_X(x_2) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$
$\gamma_Y(x_3) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$, $\gamma_Y(x_4) = \emptyset$.
Let Γ_Y be an FP-fuzzy soft set over R defined by
$Y = \{0.5/x_1, 0/x_2, 0/x_3, 0.5/x_4\}$,
$\gamma_Y(x_1) = \{\overline{0}/0.4, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$, $\gamma_Y(x_2) = \emptyset$,
$\gamma_Y(x_3) = \emptyset$, $\gamma_Y(x_4) = \{\overline{0}/0.4, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$.
It is clear that Γ_X and Γ_Y are FP-equivalent fuzzy soft rings over R and $X \cap Y = \emptyset$.

We can see that
$\mu_{(X\cup Y)}(x_1)=\mu_{(X\cup Y)}(x_2)$, but
$\gamma_{(X\cup Y)}(x_1)=\{\overline{0}/0.4,\overline{1}/0.2,\overline{2}/0.3,\overline{3}/0.2\}\neq\gamma_{(X\cup Y)}(x_2)$
$=\{\overline{0}/0.6,\overline{1}/0.4,\overline{2}/0.5,\overline{3}/0.4\}$.
Then $\Gamma_X\widetilde{\cup}\Gamma_Y$ is not an FP-equivalent fuzzy soft ring over R.

Theorem 4.8 *Let $\Gamma_X=\{(\mu_X(x)/x,\gamma_X(x)) : x\in A,\gamma_X(x)\in F(R),\mu_X(x)\in[0,1]\}$ be an FP-equivalent fuzzy soft ring over R and $\Gamma_Y=\{(\mu_Y(x)/x,\gamma_Y(x)) : x\in B, \gamma_Y(x)\in F(K),\mu_Y(x)\in[0,1]\}$ be an FP-fuzzy soft set over ring K. If Γ_X is FP-fuzzy soft homomorphic to Γ_Y, then Γ_Y is an FP-equivalent fuzzy soft ring over K.*

Proof. Let (f,g) be an FP-fuzzy soft homomorphism from Γ_X to Γ_Y. Since Γ_X is an FP-equivalent fuzzy soft ring over R, $\gamma_X(x_1)=\gamma_X(x_2)$ for all $x_1,x_2\in A$, $\mu_X(x_1)=\mu_X(x_2)$. Now, for all $y_1,y_2\in B$ and $\mu_Y(y_1)=\mu_Y(y_2)$, then there exist $x_1,x_2\in A$ such that $g(x_1)=y_1$, $g(x_2)=y_2$. Since $\mu_Y(y_1)=\mu_Y(g(x_1))=\mu_X(x_1)$ and $\mu_Y(y_2)=\mu_Y(g(x_2))=\mu_X(x_2)$, then $\mu_X(x_1)=\mu_X(x_2)$. Hence, $\gamma_Y(y_1)=\gamma_Y(g(x_1))=f(\gamma_X(x_1))=f(\gamma_X(x_2))=\gamma_Y(g(x_2))=\gamma_Y(y_2)$ and Γ_Y must be an FP-soft fuzzy ring over K as well. ∎

5. FP-increasing(decreasing) fuzzy soft rings

Definition 5.1 Let $\Gamma_X=\{(\mu_X(x)/x,\gamma_X(x)) : x\in E,\gamma_X(x)\in F(R),\mu_X(x)\in[0,1]\}$ be an FP-fuzzy soft ring over R. Then Γ_X is said to be FP-increasing fuzzy soft ring over R if, for any $x,y\in E$, $\mu_X(x)\leq\mu_X(y)$, we have $\gamma_X(x)\subseteq\gamma_X(y)$, and Γ_X is said to be FP-decreasing fuzzy soft ring over R if, for any $x,y\in E$, $\mu_X(x)\leq\mu_X(y)$, we have $\gamma_X(x)\supseteq\gamma_X(y)$.

Example 5.2 Let $R=Z_4$, $E=\{x_1,x_2,x_3,x_4\}$ and X be a fuzzy set over E defined by
$X=\{0.6/x_1,0.5/x_2,0.3/x_3,0.2/x_4\}$,
$\gamma_X(x_1)=\{\overline{0}/1,\overline{1}/0.6,\overline{2}/0.7,\overline{3}/0.6\}$, $\gamma_X(x_2)=\{\overline{0}/0.8,\overline{1}/0.5,\overline{2}/0.6,\overline{3}/0.5\}$,
$\gamma_X(x_3)=\{\overline{0}/0.8,\overline{1}/0.5,\overline{2}/0.6,\overline{3}/0.5\}$, $\gamma_X(x_4)=\{\overline{0}/0.7,\overline{1}/0.3,\overline{2}/0.5,\overline{3}/0.3\}$,
It is clearly that F_X is an FP-increasing fuzzy soft ring over R.

Notation 5.3 If $\Gamma_X=\{\mu_X(x)/x,\gamma_X(x)) : x\in E,\gamma_X(x)\in F(R),\mu_X(x)\in[0,1]\}$ and $\Gamma_Y=\{(\mu_Y(x)/x,\gamma_Y(x)) : x\in E,\gamma_Y(x)\in F(R),\mu_Y(x)\in[0,1]\}$ are FP-increasing fuzzy soft rings over ring R, $\Gamma_X\widetilde{\cap}\Gamma_Y$ is not always an FP-increasing fuzzy soft ring over R.

Example 5.4 Let $R=Z_4$, $E=\{x_1,x_2\}$. Let X be a fuzzy set over E defined by
$X=\{0.5/x_1,0.4/x_2\}$,
$\gamma_X(x_1)=\{\overline{0}/1,\overline{1}/0.6,\overline{2}/0.7,\overline{3}/0.6\}$, $\gamma_X(x_2)=\{\overline{0}/0.7,\overline{1}/0.2,\overline{2}/0.3,\overline{3}/0.2\}$.
Let Y be a fuzzy set over E defined by
$Y=\{0.1/x_1,0.9/x_2\}$,

$\gamma_Y(x_1) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$, $\gamma_Y(x_2) = \{\overline{0}/0.8, \overline{1}/0.5, \overline{2}/0.6, \overline{3}/0.5\}$.
It is clear that Γ_X and Γ_Y are FP-increasing fuzzy soft rings over R. We can see that
$X \cap Y = \{0.1/x_1, 0.4/x_2\}$, but
$\gamma_{(X\cap Y)}(x_1) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$,
$\gamma_{(X\cap Y)}(x_2) = \{\overline{0}/0.7, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$.
Then $\Gamma_X \widetilde{\cap} \Gamma_Y$ is not an FP-increasing fuzzy soft ring over R.

Notation 5.5 If $\Gamma_X = \{\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ and $\Gamma_Y = \{(\mu_Y(x)/x, \gamma_Y(x)) : x \in E, \gamma_Y(x) \in F(R), \mu_Y(x) \in [0,1]\}$ are FP-increasing fuzzy soft rings over ring R with $X \cap Y = \emptyset$, $\Gamma_X \widetilde{\cup} \Gamma_Y$ is not always an FP-increasing fuzzy soft ring over R.

Example 5.6 Let $R = Z_4$, $E = \{x_1, x_2\}$. Let X be a fuzzy set over E defined by
$X = \{0/x_1, 0.5/x_2\}$,
$\gamma_X(x_1) = \emptyset$, $\gamma_X(x_2) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$.
Let Y be a fuzzy set over E defined by
$Y = \{0.9/x_1, 0/x_2\}$,
$\gamma_Y(x_1) = \{\overline{0}/0.5, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$, $\gamma_Y(x_2) = \emptyset$.
It is clear that F_X and F_Y are FP-increasing fuzzy soft rings over R and $X \cap Y = \emptyset$. We can see that
$(X \cup Y) = \{0.9/x_1, 0.5/x_2\}$, but
$\gamma_{(X\cup Y)}(x_1) = \{\overline{0}/0.5, \overline{1}/0.2, \overline{2}/0.3, \overline{3}/0.2\}$,
$\gamma_{(X\cup Y)}(x_2) = \{\overline{0}/0.6, \overline{1}/0.4, \overline{2}/0.5, \overline{3}/0.4\}$.
Then $\Gamma_X \widetilde{\cup} \Gamma_Y$ is not an FP-increasing fuzzy soft ring over R.

Theorem 5.7 *Let $\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in A, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ be an FP-increasing fuzzy soft ring over R and $\Gamma_Y = \{(\mu_Y(x)/x, \gamma_Y(x)) : x \in B, \gamma_Y(x) \in F(K), \mu_Y(x) \in [0,1]\}$ be an FP-fuzzy soft set over ring K. If Γ_X is FP-fuzzy soft homomorphic to Γ_Y, then Γ_Y is an FP-increasing fuzzy soft ring over K.*

Proof. Let (f, g) be an FP-fuzzy soft homomorphism from Γ_X to Γ_Y. Since Γ_X is an FP-increasing fuzzy soft ring over R, for all $x_1, x_2 \in A, \mu_X(x_1) \leq \mu_X(x_2)$, $\gamma_X(x_1) \subseteq \gamma_X(x_2)$. Now, for all $y_1, y_2 \in B$ and $\mu_Y(y_1) \leq \mu_Y(y_2)$, then there exist $x_1, x_2 \in A$ such that $g(x_1) = y_1$, $g(x_2) = y_2$. Since $\mu_Y(y_1) = \mu_Y(g(x_1)) = \mu_X(x_1)$ and $\mu_Y(y_2) = \mu_Y(g(x_2)) = \mu_X(x_2)$, then $\mu_X(x_1) \leq \mu_X(x_2)$. Hence, $\gamma_Y(y_1) = \gamma_Y(g(x_1)) = f(\gamma_X(x_1)) \subseteq f(\gamma_X(x_2)) = \gamma_Y(g(x_2)) = \gamma_Y(y_2)$ and Γ_Y must be an FP-increasing fuzzy soft ring over K as well. ∎

Corollary 5.8 *If $\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ is both a FP-equivalent fuzzy soft ring and FP-increasing fuzzy soft ring over ring R, then $\Gamma_{\bar{X}} = \{(\mu_{\bar{X}}(\bar{x})/\bar{x}, \gamma_{\bar{X}}(\bar{x})) : \bar{x} \in \bar{E}, \gamma_{\bar{X}}(\bar{x}) \in F(R), \mu_{\bar{X}}(\bar{x}) \in [0,1]\}$ is an FP-increasing fuzzy soft ring over ring R.*

6. Aggregate fuzzy subrings

In [3], N. Çağman et al. defined an aggregate fuzzy set of an FP-fuzzy soft set. They also defined $FPFS$-aggregation operator that produced an aggregate fuzzy set from an FP-fuzzy soft set and its fuzzy parameter set.

Definition 6.1 [3] Let $\Gamma_X \in FPFS(U)$. Then $FPFS$-aggregation operator, denoted by $FPFS_{agg}$ is defined by

$$FPFS_{agg} : F(E) \times FPFS(U) \longrightarrow F(U),$$

$$FPFS_{agg}(X, \Gamma_X) = \Gamma_X^*$$

where

$$\Gamma_X^* = \{\mu_{\Gamma_X^*}(u)/u : u \in U\}$$

which is a fuzzy set over U. The value Γ_X^* is called aggregate fuzzy set of the Γ_X. Here the membership degree $\mu_{\Gamma_X^*}(u)$ of u is defined as follows

$$\mu_{\Gamma_X^*}(u) = \frac{1}{\mid E \mid} \sum_{x \in E} \mu_X(x) \mu_{\gamma_X(x)}(u)$$

where $\mid E \mid$ is the cardinality of E.

Theorem 6.2 *Let* $\Gamma_X = \{(\mu_X(x)/x, \gamma_X(x)) : x \in E, \gamma_X(x) \in F(R), \mu_X(x) \in [0,1]\}$ *be an* FP*-fuzzy soft ring over* R*. Then the aggregate fuzzy set* Γ_X^* *of* Γ_X *is a fuzzy subring of* R*.*

Proof. For any $x \in E$, $\gamma_X(x)$ is a fuzzy subring of R. Then for all $r, s \in R$, $\mu_{\gamma_X(x)}(r-s) \geq \min\{\mu_{\gamma_X(x)}(r), \mu_{\gamma_X(x)}(s)\}$ and $\mu_{\gamma_X(x)}(rs) \geq \min\{\mu_{\gamma_X(x)}(r), \mu_{\gamma_X(x)}(s)\}$. Then

$$\begin{aligned} \mu_{\Gamma_X^*}(r-s) &= \frac{1}{\mid E \mid} \sum_{x \in E} \mu_X(x) \mu_{\gamma_X(x)}(r-s) \\ &\geq \min\left\{ \frac{1}{\mid E \mid} \sum_{x \in E} \mu_X(x) \mu_{\gamma_X(x)}(r), \frac{1}{\mid E \mid} \sum_{x \in E} \mu_X(x) \mu_{\gamma_X(x)}(s) \right\} \\ &= \min\{\mu_{\Gamma_X^*}(r), \mu_{\Gamma_X^*}(s)\}. \end{aligned}$$

In the same way, we can obtain $\mu_{\Gamma_X^*}(rs) \geq \min\{\mu_{\Gamma_X^*}(r), \mu_{\Gamma_X^*}(s)\}$. Which is to say that Γ_X^* is a fuzzy subring of R. ∎

Notation 6.3 Above Γ_X^* is called an aggregate fuzzy subring of FP-fuzzy soft ring Γ_X.

Example 6.4 Let R be a full matrix ring, written by M_n, let A be an upper triangular matrix ring and B a symmetrical matrix ring. And let $E = \{a, b\}$, the parameters a, b stand for "upper triangular" and "symmetrical", respectively. And X be a fuzzy set over E defined by

$$\mu_X(x) = \begin{cases} 1, & x = a, \\ 0.8, & x = b. \end{cases}$$

Let γ_X be defined by

$$\mu_{\gamma_X(a)}(r) = \begin{cases} 0, & r \text{ is not an upper triangular matrix,} \\ 1, & r \text{ is an upper triangular matrix.} \end{cases}$$

$$\mu_{\gamma_X(b)}(r) = \begin{cases} 0, & r \text{ is not symmetrical,} \\ 1, & r \text{ is symmetrical.} \end{cases}$$

It is clear that Γ_X is an FP-fuzzy soft ring over M_n. The aggregate fuzzy set can be found as

$$\Gamma^*_X(m) = \begin{cases} 0.9, & \text{if } m \in A \cap B\ , \\ 0.5, & \text{if } m \in A - B, \\ 0.4, & \text{if } m \in B - A, \\ 0, & \text{otherwise.} \end{cases}$$

We can verify that Γ^*_X is a fuzzy ring of M_n.

Notation 6.5 Let R be a subring of M_n, if Γ_X is defined as in Example 6.4, then R is a diagonal matrix ring if and only if the aggregate fuzzy subring of Γ_X is $\Gamma^*_X = 0.9$.

Notation 6.6 Let R be a subring of M_n, if Γ_X is defined as in Example 6.4, then the Γ^*_X is called a fuzzy diagonal subring of R related to the FP-fuzzy soft ring Γ_X.

Remark 6.7 We can define another fuzzy diagonal subring of R related to another FP-fuzzy soft ring.

Acknowledgement. This research is partially supported by a grant of National Natural Science Foundation of China (11461025) and Innovation Term of Hubei University for Nationalities (MY2014T002).

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Accepted: 20.09.2014