INTUITIONISTIC FUZZY α-IRRESOLUTE FUNCTIONS

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Abstract. In this paper the concept of intuitionistic fuzzy α-irresolute functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. We also study relationship between this function with other existing functions.

Keywords: intuitionistic fuzzy α-open set, intuitionistic fuzzy α-continuous, intuitionistic fuzzy α-irresolute.

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1. Introduction

Ever since the introduction of fuzzy sets by L.A. Zadeh [10], the fuzzy concept has invaded almost all branches of mathematics. The concept of fuzzy topological spaces was introduced and developed by C.L. Chang [2]. Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] introduced the intuitionistic fuzzy topological spaces. In this paper, we have introduced the concept of intuitionistic fuzzy α-irresolute functions and studied their properties. Also, we have given characterizations of intuitionistic fuzzy α-irresolute functions. We also study the relationship between this function with other existing functions.

2. Preliminaries

Definition 2.1. [1] Let X be a nonempty fixed set and I the closed interval $[0, 1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\},$$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of nonmembership (namely) $\nu_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. [1] Let A and B are IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;

(ii) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$;

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$;

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

We will use the notation $A = \{\langle x, \mu_A, \nu_A\rangle \mid x \in X\}$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$.

Definition 2.3. [3] $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$.

Let $\alpha, \beta \in [0,1]$ such that $\alpha+\beta \leq 1$. An intuitionistic fuzzy point (IFP) $p_{(\alpha,\beta)}$ is an intuitionistic fuzzy set defined by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

Let X and Y are two non-empty sets and $f : (X,\tau) \to (Y,\sigma)$ be a function.

If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \mid y \in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by

$$f^{-1}(B) = \{\left\langle x, f^{-1}(\mu_B(x)), f^{-1}(\nu_B(x))\right\rangle \mid x \in X\}.$$

Since μ_B, ν_B are fuzzy sets, we explain that $f^{-1}(\mu_B(x)) = \mu_B(f(x))$.

Definition 2.4. [3] An intuitionistic fuzzy topology(IFT) in Coker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$;

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this paper, by (X,τ) or, simply, by X we will denote the intuitionistic fuzzy topological space (IFTS). Each IFS which belongs to τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set(IFCS) in X.

Definition 2.5. [3] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ be an IFS in X. Then, the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by

(i) $\mathrm{cl}(A) = \cap\{C : C \text{ is an IFCS in } X \text{ and } C \supseteq A\}$;

(ii) $\mathrm{int}(A) = \cup\{D : D \text{ is an IFOS in } X \text{ and } D \subseteq A\}$.

It can be also shown that $\mathrm{cl}(A)$ is an IFCS, $\mathrm{int}(A)$ is an IFOS in X and A is an IFCS in X if and only if $\mathrm{cl}(A) = A$; A is an IFOS in X if and only if $\mathrm{int}(A) = A$.

Proposition 2.1. [3] *Let (X, τ) be an IFTS and A, B be IFSs in X. Then, the following properties hold:*

(i) $\mathrm{cl}\overline{A} = \overline{(\mathrm{int}(A))}, \mathrm{int}\overline{(A)} = \overline{(\mathrm{cl}(A))}$;

(ii) $\mathrm{int}(A) \subseteq A \subseteq \mathrm{cl}(A)$.

Definition 2.6. [5] An IFS A in an IFTS X is called an intuitionistic fuzzy pre open set (IFPOS) if $A \subseteq \mathrm{int}(\mathrm{cl}\, A)$. The complement of an IFPOS A in IFTS X is called an intuitionistic fuzzy preclosed (IFPCS) in X.

Definition 2.7. [5] An IFS A in an IFTS X is called an intuitionistic fuzzy α-open set (IFαOS) if and only if $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}\, A))$. The complement of an IFαOS A in X is called intuitionistic fuzzy α-closed (IFαCS) in X.

Definition 2.8. [5] An IFS A in an IFTS X is called an intuitionistic fuzzy semi open set (IFSOS) if and only if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$. The complement of an IFSOS A in X is called intuitionistic fuzzy semi closed(IFSCS) in X.

Definition 2.9. Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

(i) intuitionistic fuzzy continuous if and only if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y [5];

(ii) intuitionistic fuzzy α-continuous if and only if $f^{-1}(B)$ is an IFαOS in X, for each IFOS B in Y [5];

(iii) intuitionistic fuzzy pre continuous if and only if $f^{-1}(B)$ is an IFPOS in X, for each IFOS B in Y [5];

(iv) intuitionistic fuzzy semi continuous if and only if $f^{-1}(B)$ is an IFSOS in X, for each IFOS B in Y [5].

Definition 2.10. [9] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy α-closure and intuitionistic fuzzy α-interior of A are defined by

(i) $\alpha\mathrm{cl}(A) = \bigcap\{C : C \text{ is an IF}\alpha\text{CS in } X \text{ and } C \supseteq A\}$;

(ii) $\alpha\mathrm{int}(A) = \bigcup\{D : D \text{ is an IF}\alpha\text{OS in } X \text{ and } D \subseteq A\}$.

Definition 2.11. Let f be a mapping from an IFTS of X into an IFTS of Y. The mapping f is called intuitionistic fuzzy strongly α-continuous if and only if $f^{-1}(B)$ is an IFαOS in X, for each IFSOS B in Y.

3. Intuitionistic fuzzy α-irresolute functions

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ from an intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,σ) is said to be intuitionistic fuzzy α-irresolute(IF α-irresolute) if $f^{-1}(B)$ is an IFαOS in (X,τ) for each IFαOS B in (Y,σ).

$$\begin{array}{ccccc}
\text{IF strongly } \alpha\text{-continuous} & & \longrightarrow & & \text{IF } \alpha\text{-continuous} \\
& \searrow & & \nearrow & \\
& & \text{IF } \alpha\text{-irresolute} & & \\
& \swarrow & & \searrow & \\
\text{IF semi continuous} & & & & \text{IF pre continuous}
\end{array}$$

Proposition 3.1. *Every intuitionistic fuzzy α-irresolute is an intuitionistic fuzzy pre continuous.*

Proof. Follows from the definitions. ∎

However, the converse of the above Proposition 3.1 need not to be true, as shown by the following example.

Example 3.1. Let $X = \{a, b\}$, $Y = \{c, d\}$, $\tau = \{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B\}$, where

$$\begin{aligned}
A &= \left\{\left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\right\rangle;\ x \in X\right\}, \\
B &= \left\{\left\langle y, \left(\frac{c}{0.2}, \frac{d}{0.4}\right), \left(\frac{c}{0.6}, \frac{d}{0.5}\right)\right\rangle\ \ y \in Y\right\}.
\end{aligned}$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d$, $f(b) = c$. B is an IFOS in (Y, σ). $f^{-1}(B) = \left\{\left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right)\right\rangle;\ x \in X\right\}$ is an IFPOS in (X, τ), since $\text{int}(\text{cl}(f^{-1}(B))) = 1_\sim$, and $f^{-1}(B) \subseteq \text{int}(\text{cl}(f^{-1}(B)))$. Hence, f is an IF pre continuous. B is an IFαOS in (Y, σ) and $\text{int}(\text{cl}(\text{int}(f^{-1}(B)))) = 0_\sim$, $f^{-1}(B) \nsubseteq \text{int}(\text{cl}(\text{int}(f^{-1}(B))))$. Hence, $f^{-1}(B)$ is not IFαOS in (X, τ) which implies f is not IF α-irresolute function.

Proposition 3.2. *Every intuitionistic fuzzy α-irresolute is an intuitionistic fuzzy α-continuous.*

Proof. Follows from the definitions. ∎

However, the converse of the above Proposition 3.2 need not to be true, as shown by the following example.

Example 3.2. Let $X = \{a, b, c\} = Y$, $\tau = \{0_\sim, AB, A \cup B, A \cap B, 1_\sim\}$, $\sigma = \{0_\sim, 1_\sim, C\}$, where

$$
\begin{aligned}
A &= \left\{\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.7}, \frac{c}{0.4}\right)\right\rangle;\ x \in X\right\}, \\
B &= \left\{\left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.3}\right) \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.7}\right)\right\rangle;\ x \in X\right\} \\
C &= \left\{\left\langle y, \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}\right)\right\rangle;\ y \in Y\right\} \\
D = &= \left\{\left\langle y, \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right)\right\rangle;\ y \in Y\right\}
\end{aligned}
$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. C is an IFOS in (Y, σ). $f^{-1}(C) = \left\{\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}\right)\right\rangle;\ x \in X\right\}$ and $\mathrm{int}(\mathrm{cl}(\mathrm{int} f^{-1}(C))) = A \cup B$. Thus $f^{-1}(C) \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int} f^{-1}(C)))$. Hence $f^{-1}(C)$ is IFαOS in X, which implies f is IF α-continuous. D is an IFS in Y and $D \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}\, D)) = 1_\sim$. Hence, D is IFαOS in Y. $f^{-1}(D) = \left\{\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}\right)\right\rangle;\ x \in X\right\}$ and $\mathrm{int}(\mathrm{cl}(\mathrm{int} f^{-1}(D))) = A \cup B$. Thus, $f^{-1}(D) \nsubseteq \mathrm{int}(\mathrm{cl}(\mathrm{int} f^{-1}(D)))$. Hence, $f^{-1}(D)$ is not IFαOS in X. So, f is not IF α-irresolute function.

Proposition 3.3. *Every intuitionistic fuzzy α-continuous is an intuitionistic fuzzy pre continuous.*

Proof. Follows from the definitions. ∎

However, the converse of the above Proposition 3.3 need not to be true, as shown by the following example.

Example 3.3. Let $X = \{a, b\}$, $Y = \{c, d\}$, $\tau = \{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B\}$, where

$$
\begin{aligned}
A &= \left\{\left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\right\rangle;\ x \in X\right\}, \\
B &= \left\{\left\langle y, \left(\frac{c}{0.2}, \frac{d}{0.4}\right), \left(\frac{c}{0.6}, \frac{d}{0.5}\right)\right\rangle;\ y \in Y\right\}.
\end{aligned}
$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d$, $f(b) = c$. B is an IFOS in (Y, σ). $f^{-1}(B) = \left\{\left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right)\right\rangle;\ x \in X\right\}$ is an IFPOS in X since $\mathrm{int}(\mathrm{cl}(f^{-1}(B))) = 1_\sim$, and $f^{-1}(B) \subseteq \mathrm{int}(\mathrm{cl}(f^{-1}(B)))$. Hence, f is an IF pre continuous. $f^{-1}(B)$ is not IFαOS in X since $f^{-1}(B) \nsubseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(B)))) = 0_\sim$. Hence, f is not IFα-continuous function.

Proposition 3.4. *Every intuitionistic fuzzy α-irresolute is an intuitionistic fuzzy semi continuous.*

Proof. Follows from the definitions. ∎

However, the converse of the above Proposition 3.4 need not to be true, as shown by the following example.

Example 3.4. Let $X = \{a, b\}$, $Y = \{c, d\}$, $\tau = \{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B\}$, where

$$\begin{aligned} A &= \left\{\left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}\right)\right\rangle ;\ x \in X\right\}, \\ B &= \left\{\left\langle y, \left(\frac{c}{0.4}, \frac{d}{0.5}\right), \left(\frac{c}{0.5}, \frac{d}{0.5}\right)\right\rangle ;\ y \in Y\right\}. \end{aligned}$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d$, $f(b) = c$. B is an IFOS in (Y, σ). $f^{-1}(B) = \left\{\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right)\right\rangle ;\ x \in X\right\}$ is an IFSOS in (X, τ) since $\mathrm{cl}(\mathrm{int}(f^{-1}(B))) = \overline{A}$. Hence $f^{-1}(B) \subseteq \mathrm{cl}(\mathrm{int}(f^{-1}(B)))$, which implies f is an IF Semi continuous. B is an IFOS in Y and B is an IFαOS in Y since $B \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(B))) = B$. $f^{-1}(B) = \left\{\left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right)\right\rangle ;\ x \in X\right\}$ $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(B)))) = A$ and $f^{-1}(B) \nsubseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(B))))$ which implies $f^{-1}(B)$ is not IFαOS in Y. Hence f is not IF α-irresolute function.

Proposition 3.5. *Every intuitionistic fuzzy strongly α-continuous is an intuitionistic fuzzy α-irresolute.*

Proof. Follows from the definitions. ∎

However, the converse of the above Proposition 3.5 is need not be true, in general as shown by the following example.

Example 3.5. Let $X = \{a, b\}$, $Y = \{c, d\}$, $\tau = \{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B\}$ where

$$\begin{aligned} A &= \left\{\left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right)\right\rangle ;\ x \in X\right\}, \\ B &= \left\{\left\langle y, \left(\frac{c}{0.4}, \frac{d}{0.2}\right), \left(\frac{c}{0.4}, \frac{d}{0.3}\right)\right\rangle ;\ y \in Y\right\}, \\ C &= \left\{\left\langle y, \left(\frac{c}{0.4}, \frac{d}{0.3}\right), \left(\frac{c}{0.4}, \frac{d}{0.2}\right)\right\rangle ;\ y \in Y\right\}. \end{aligned}$$

Define an intuitionistic fuzzy mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d$, $f(b) = c$. B is an IFOS in (Y,σ) and $\mathrm{int}(\mathrm{cl}(\mathrm{int}(B))) = B$. Hence $B \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(B)))$. Thus B is an IFαOS in (Y,σ). $f^{-1}(B) = \left\{\left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right)\right\rangle ; x \in X\right\}$ is an IFαOS in (X,τ) since $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(B)))) = A$. Hence $f^{-1}(B) \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(B))))$ which implies f is an IF α-irresolute. C is an IFS in Y. Also C is an IFSOS in Y since $C \subseteq \mathrm{cl}(\mathrm{int}(C)) = \overline{B}$. $f^{-1}(C) = \left\{\left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}\right)\right\rangle ;\ x \in X\right\}$ $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(C)))) = A$ and $f^{-1}(C) \nsubseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(C))))$ which implies $f^{-1}(C)$ is not IFαOS in Y. Hence f is not IF strongly α-continuous.

Proposition 3.6. *Every intuitionistic fuzzy strongly α-continuous is an intuitionistic fuzzy α-continuous.*

Proof. Follows from the definitions. ∎

However the converse of the above Proposition 3.6 is need not be true, as shown by the following example.

Example 3.6. Let $X = \{a,b\}$, $Y = \{c,d\}$, $\tau = \{0_\sim, 1_\sim, A\}$, $\sigma = \{0_\sim, 1_\sim, B\}$, where

$$\begin{aligned} A &= \left\{\left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}\right)\right\rangle;\ x \in X\right\}, \\ B &= \left\{\left\langle y, \left(\frac{c}{0.4}, \frac{d}{0.2}\right), \left(\frac{c}{0.4}, \frac{d}{0.3}\right)\right\rangle;\ y \in Y\right\}, \\ C &= \left\{\left\langle y, \left(\frac{c}{0.4}, \frac{d}{0.3}\right), \left(\frac{c}{0.4}, \frac{d}{0.2}\right)\right\rangle;\ y \in Y\right\}. \end{aligned}$$

Define an intuitionistic fuzzy mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = d$, $f(b) = c$. B is an IFOS in (Y,σ). $f^{-1}(B) = \{\langle x, (\frac{a}{0.2}, \frac{b}{0.4}), (\frac{a}{0.3}, \frac{b}{0.4})\rangle;\ x \in X\}$ is an IFαOS in (X,τ) since int(cl(int($f^{-1}(B)$))) $= A$. Hence $f^{-1}(B) \subseteq$ int(cl(int($f^{-1}(B)$))) which implies f is an IFα-continuous. C is an IFS in Y. Also C is an IFSOS in Y since $C \subseteq$ cl(int(C)) $= \overline{B}$. $f^{-1}(C) = \{\langle x, (\frac{a}{0.3}, \frac{b}{0.4}), (\frac{a}{0.2}, \frac{b}{0.4})\rangle;\ x \in X\}$ int(cl(int($f^{-1}(C)$))) $= A$ and $f^{-1}(C) \nsubseteq$ int(cl(int($f^{-1}(C)$))) which implies $f^{-1}(C)$ is not IFαOS in Y. Hence f is not IF strongly α-continuous.

Theorem 3.1. *If $f : (X,\tau) \to (Y,\sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following are equivalent.*

(a) *f is IF α-irresolute.*

(b) *$f^{-1}(B)$ is IFαCS in X for each IFαCS B in Y.*

(c) *$f(\alpha\mathrm{cl}\,A) \subseteq \alpha\,\mathrm{cl}\,f(A)$ for each IFS A in X.*

(d) *$\alpha\,\mathrm{cl}\,f^{-1}(B) \subseteq f^{-1}(\alpha\,\mathrm{cl}\,B)$ for each IFS B in Y.*

(e) $f^{-1}(\alpha\mathrm{int}\,B) \subseteq \alpha\,\mathrm{int}\,f^{-1}(B)$ for each IFS B in Y.

Proof. (a)⇒(b): It can be proved by using the complement and the definition of IFα-irresolute. Let B be IFαCS in Y, then 1-B is IFαOS in Y. Since f is IFα-irresolute, $f^{-1}(1 - B) =$ 1-$f^{-1}(B)$ is IFαOS in X. Hence $f^{-1}(B)$ is IFαCS in X. Thus (a)⇒(b) is proved.

(b)⇒(c): Let A be IFS in X. Then A$\subseteq f^{-1}(f(A)) \subseteq f^{-1}(\alpha clf(A))$.
$\alpha clf(A)$ is IFαCS in Y, by (b) $f^{-1}(\alpha clf(A))$ is an IFαCS in X.
αcl(A) $\subseteq f^{-1}(\alpha clf(A))$ and $f(\alpha cl(A)) \subseteq f(f^{-1}(\alpha clf(A))) = \alpha clf(A)$.
Thus $f(\alpha cl(A)) \subseteq \alpha clf(A)$. Hence (b)⇒(c) is proved.

(c)⇒(d): For any IFS B in Y, let $f^{-1}(B) =$ A; by (c), $f(\alpha\text{cl}f^{-1}(B)) \subseteq \alpha cl f(f^{-1}(B))$ $\subseteq \alpha cl(B)$ and $(\alpha\text{cl}f^{-1}(B)) \subseteq f^{-1}(f(\alpha cl f^{-1}(B))) \subseteq f^{-1}(\alpha cl B)$. Thus $(\alpha cl f^{-1}(B))$ $\subseteq f^{-1}(\alpha cl B)$. Hence (c)⇒(d) is proved.

(d)⇒(e): We know that αint(B) = $\overline{\alpha cl(\overline{B})}$
$f^{-1}(\alpha\text{intB}) = f^{-1}\overline{(\alpha cl(\overline{B}))} = \overline{(f^{-1}(\alpha cl(\overline{B})))} \subseteq \overline{(\alpha cl(f^{-1}(\overline{B}))} = \overline{\overline{\alpha int(\overline{f^{-1}(B)})}} \subseteq$ αint(f^{-1}(B)) Hence (d)⇒(e) is proved.

(e)⇒(a):Let B be any IFαOS in Y. Then B = αint(B).
f^{-1}(αintB)= f^{-1}(B) $\subseteq$ αint(f^{-1}(B)).
By definition, f^{-1}(B) $\supseteq$ αint(f^{-1}(B)). So f^{-1}(B) = αint(f^{-1}(B)). Thus f^{-1}(B) is an IFαOS in X which implies f is IF α-irresolute. Thus (e)⇒(a) is proved. ∎

4. Properties of intuitionistic fuzzy α-irresolute functions

Lemma 4.1. [3] *Let f:X→Y be a mapping, and A_α be a family of IF sets of Y. Then*

(a) $f^{-1}(\bigcup A_\alpha) = \cup f^{-1}(A_\alpha)$

(b) $f^{-1}(\cap A_\alpha) = \cap f^{-1}(A_\alpha)$.

Lemma 4.2. [6] *Let $f : X_i \to Y_i$ be a mapping and A,B are IFS's of Y_1 and Y_2 respectively then $(f_1 \times f_2)^{-1}(A \times B) = {f_1}^{-1}(A) \times {f_2}^{-1}(B)$.*

Lemma 4.3. [6] *Let $g : X \to X \times Y$ be a graph of a mapping $f : (X, \tau) \to (Y, \sigma)$. If A and B are IFS's of X and Y respectively, then $g^{-1}(1_\sim \times B) = (1_\sim \cap f^{-1}(B))$.*

Lemma 4.4. [6] *Let X and Y be intuitionistic fuzzy topological spaces, then (X, τ) is product related to (Y, σ) if for any IFS C in X, D in Y whenever $\overline{A} \not\supseteq C$, $\overline{B} \not\supseteq D$ implies $\overline{A} \times 1_\sim \cup 1_\sim \times \overline{B} \supseteq C \times D$ there exists $A_1 \in \tau$, $B_1 \in \sigma$ such that $\overline{A_1} \supseteq C$ and $\overline{B_1} \supseteq D$ and $\overline{A_1} \times 1_\sim \cup 1_\sim \times \overline{B_1} = \overline{A} \times 1_\sim \cup 1_\sim \times \overline{B}$.*

Lemma 4.5. *Let X and Y be intuitionistic fuzzy topological spaces such that X is product related to Y. Then the product A×B of an IFαOS A in X and an IFαOS B in Y is an IFαOS in fuzzy product spaces X×Y.*

Theorem 4.1. *Let $f : X \to Y$ be a function and assume that X is product related to Y. If the graph $g : X \to X \times Y$ of f is IF α-irresolute then so is f.*

Proof. Let B be IFαOS in Y. Then by Lemma 4.3 f^{-1}(B) = $1_\sim \cap f^{-1}$(B) = g^{-1} ($1_\sim$ × B). Now, $1_\sim \times B$ is IFαOS in $X \times Y$. Since g is IF α-irresolute, $g^{-1}(1_\sim \times B)$ is IFαOS in X. Hence $f^{-1}(B)$ is IFαOS in X. Thus f is IF α-irresolute function. ∎

Theorem 4.2. *If a function $f : X \to \Pi Y_i$ is IF α-irresolute, then $P_i \circ f : X \to Y_i$ is IF α-irresolute, where P_i is the projection of ΠY_i onto Y.*

Proof. Let B_i be any IFαOS of Y_i. Since P_i is IF continuous and IFOS, it is IFαOS. Now P_i: $\Pi Y_i \to Y_i$; $P_i{}^{-1}(B_i)$ is IFαOS in Π Y_i. Therefore, P_i is IF α-irresolute function. Now $(P_i \circ f)^{-1}(B_i) = f^{-1}$ $(P_i{}^{-1}(B_i))$, since f is IF α-irresolute and $P_i{}^{-1}(B_i)$ is IFαOS, f^{-1} $(P_i{}^{-1}(B_i))$ is IFαOS. Hence $(P_i \circ f)$ is IF α-irresolute.

Theorem 4.3. *If $f_i : X_i \to Y_i$, $(i = 1, 2)$ are IF α-irresolute and X_1 is product related to X_2, then $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is IF α-irresolute.*

Proof. Let $C = \cup(A_i \times B_i)$ where A_i and B_i, $i = 1, 2$ are IFα-open sets of Y_1 and Y_2 respectively. Since Y_1 is product related to Y_2, by Lemma 4.5, that C $= \cup(A_i \times B_i)$ is IFα-open of $Y_1 \times Y_2$. Using Lemmas 4.1 and 4.2 we obtain $(f_1 \times f_2)^{-1}(C) = (f_1 \times f_2)^{-1} \cup(A_i \times B_i) = \cup\ (f_1{}^{-1}(A_i) \times f_2{}^{-1}(B_i))$. Since f_1 and f_2 are IF α-irresolute, we conclude that $(f_1 \times f_2)^{-1}(C)$ is an IFαOS in $X_1 \times X_2$ and hence $f_1 \times f_2$ is IF α-irresolute function. ∎

Theorem 4.4. *A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is IF α-irresolute if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFαOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IF$\alpha$OS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.*

Proof. Let f be any IF α-irresolute mapping, $p_{(\alpha,\beta)}$ be an IFP in X and B be any IFαOS in Y such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}$(B) $= \alpha$intf^{-1}(B). Let A $= \alpha$intf^{-1}(B). Then A is an IFαOS in X which containing IFP $p_{(\alpha,\beta)}$ and $f(A) = f(\alpha int f^{-1}(B)) \subseteq f(f^{-1}(B)) = B$.

Conversely, let B be an IFαOS in Y and $p_{(\alpha,\beta)}$ be IFP in X such that $p_{(\alpha,\beta)} \in f^{-1}$(B). According to assumption there exists IFαOS A in X such that $p_{(\alpha,\beta)} \in$A and $f(A) \subseteq B$. Hence $p_{(\alpha,\beta)} \in$A $\subseteq f^{-1}$(B) and $p_{(\alpha,\beta)} \in A = \alpha$int$A \subseteq \alpha$int$f^{-1}(B)$. Since $p_{(\alpha,\beta)}$ be an arbitrary IFP and f^{-1}(B) is union of all IFP containing in f^{-1}(B) we obtain that $f^{-1}(B) = \alpha$int$f^{-1}(B)$. So, f is an IF α-irresolute mapping. ∎

Definition 4.1. [4] Let (X,τ) be an IFTS and let A be any IFS in X. Then A is called IFdense set if clA = $1_\sim$ and A is called nowhere IFdense set if int(cl(A))= $0_\sim$.

Theorem 4.5. *If a function $f : (X, \tau) \to (Y, \sigma)$ is IF α-irresolute, then $f^{-1}(A)$ is IFα-closed in X for any nowhere IFdense set A of Y.*

Proof. Let A be any nowhere IF dense set in Y. Then int(clA)= $0_\sim$. Now, 1-int(clA) = $1_\sim$.

$\Longrightarrow$ cl(1-cl(A)) = $1_\sim$ which implies cl(int(1-A)) = $1_\sim$. Since int$1_\sim$ = $1_\sim$, int(cl(int(1-A))) = int$1_\sim$ = $1_\sim$. Hence 1-A $\subseteq$ int(cl(int(1-A))) = $1_\sim$. Then 1-A is IFαOS in Y. Since f is IF α-irresolute, f^{-1}(1-A) is IFαOS in X. Hence $f^{-1}(A)$ is IFαCS in X. ∎

Theorem 4.6. *The following hold for functions $f : X \to Y$ and $g : Y \to Z$.*

(i) *If f is IF α-irresolute and g is IF α-irresolute $g \circ f$ is IF α-irresolute.*

(ii) *If f is IF α-irresolute and g is IF strongly α-continuous then g ∘f is IF strongly α-continuous.*

(iii) *If f is IF α-irresolute and g is IFα-continuous then g ∘f is IF α-irresolute.*

Proof. (i) Let B be an IFαOS in Z. Since g is IF α-irresolute, g^{-1}(B) is an IFαOS in Y. Now $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF α-irresolute, $f^{-1}(g^{-1}(B))$ is IFαOS in X. Hence $g \circ f$ is IF α-irresolute.

(ii) Let B be an IFSOS in Z. Since g is IF strongly α-continuous, g^{-1}(B) is an IFαOS in Y. Now $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF α-irresolute, $f^{-1}(g^{-1}(B))$ is IFαOS in X. Hence $g \circ f$ is IF strongly α-continuous.

(iii) Let B be an IFOS in Z. Since g is IFα-continuous, g^{-1}(B) is an IFαOS in Y. Now, $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$. Since f is IF α-irresolute, $f^{-1}(g^{-1}(B))$ is IFαOS in X. Hence $g \circ f$ is IF α-irresolute. ∎

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