NUMERICAL SOLUTION OF SERIES L–C–R EQUATION BASED ON HAAR WAVELET

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Abstract. Haar wavelet is the simplest and computer oriented tool for solving ordinary differential equations and partial differential equations. Numerical solution of Series L–C–R is very useful in many engineering branches. In this paper we shall discuss the numerical solution of series L–C–R circuit with Haar wavelet method. We shall find charge in series L–C–R circuit at different times. Two different cases show the accuracy of Haar method.

Keywords: Haar wavelet, L–C–R circuit, linear ordinary differential equation, Matlab.
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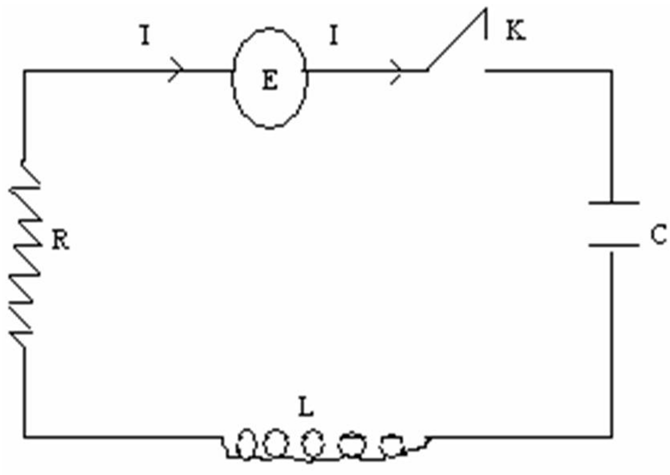
1. Introduction

Wavelet transform and wavelet analysis are recently developed as mathematical tools. The roots of wavelet find in various disciplines such as image compression, data compression, denoising data, solution of initial & boundary value problems etc. First Chen and Hsiao [1] gave a method for solving linear systems of ordinary differentia equations and partial differential equations based on haar wavelet. Haar wavelet is the simplest orthonormal wavelet with compact support. Lepik [6], [7],

[8] applied Haar wavelet in solving nonlinear integro-differential equation and partial differential equations. In recent years wavelet transform is very useful in the field of numerical approximation. Due to simplicity, Haar wavelet had become an effective tool for solving ordinary differential equations and partial differential equations.

Haar wavelet can be applied to compute the charge on the capacitors and currents as function of time. A simple electrical circuit consists of the following elements connected in series with a key K.

(i) A battery which supplied an electromotive force (E.M.F).
(ii) An inductor that has inductance L.
(iii) a resistance R and
(iv) a capacitor that has capacitance C.



Series L–C–R Circuit

The differential equation of above figure is

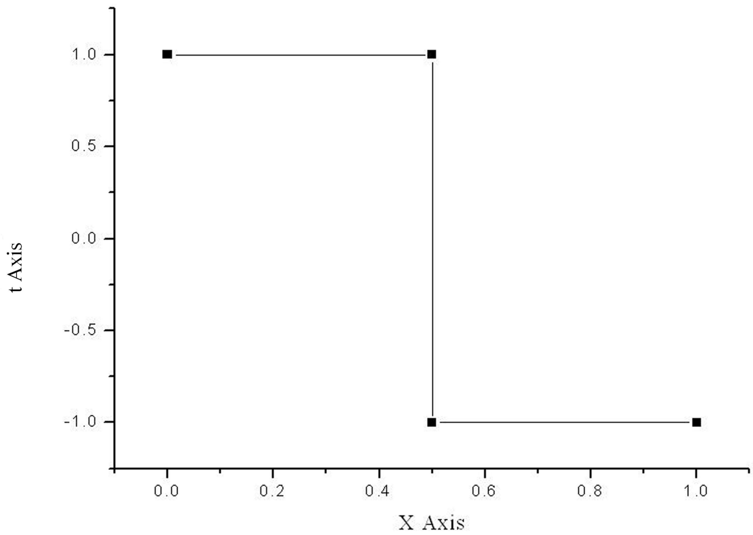
$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E \qquad Q(0) = 0, Q^{'}(0) = 0. \qquad I = \frac{dQ}{dt} \tag{1.1}$$

where Q is charge, R is resistance, L is inductance, C is capacitance, w is angular velocity and E is electromotive force.

2. Haar wavelet

The Haar wavelet was first introduced by Alfred Haar [5] in 1910. Haar wavelet is a certain sequence of rescaled "square-shaped" function which together forms a wavelet family or basis. Haar wavelet is defined for t∈ [0 1)

$$\psi(x) = \begin{cases} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$



Haar wavelet

Haar wavelet family is defined for $t \in [0\ 1)$ as follows:

$$h_i(t) = \begin{cases} 1 & \text{for } t \in [\eta_1, \eta_2) \\ -1 & \text{for } t \in [\eta_2, \eta_3) \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

where $\eta_1 = \dfrac{K}{m}$, $\eta_2 = \dfrac{K+0.5}{m}$, $\eta_3 = \dfrac{K+1}{m}$. The integer m = 2^j (j= 0, 1,. . . , J) indicates the level of the wavelet; $k = 0, 1, \ldots, m-1$ is the translation parameter. The maximal level of relation is J. The index i in (1) is calculated according to the formula $i = m + k + 1$; In the case of minimal values $m = 1, k = 0$, we have $i = 2$. The maximum value of i is $i = 2^{j+1} = M$ It is assume that the value $i = 1$ corresponding to the scaling function for which $h_1 = 1$ for $t \in [0\ 1)$.

3. Approximation of function

Any square integrable function $Q(t)$ can be expanded by haar series of infinite terms in the interval $[0, 1)$

$$Q(t) = \sum_{i=1}^{\infty} a_i h_i(t)$$

where $a_i,\ i = 1, 2...$, are haar coefficients. If $Q(t)$ be is approximate as piecewise constant during each subinterval, then $Q(t)$ will be terminated at finite M terms, that is

$$Q(t) = \sum_{i=1}^{M} a_i h_i(t).$$

4. Haar wavelet method

We follow work done by Phang Chang and Phang Piau [2], for solving n^{th} order linear ordinary differential equation

$$a_1 Q^{(n)}(t) + a_2 Q^{(n-1)}(t) + \cdots + a_n Q(t) = f(t),$$

where $t \in [A,\ B)$.

Let $M = 2^{j+1}$. The interval $[A, B)$ will be divided into M subintervals, hence $\triangle t = \dfrac{A-B}{M}$, the values at each point can be estimated by following 5 stapes.

1. Let $Q^{(n)}(t) = \sum_{i=1}^{M} a_i h_i(t)$ where h is haar wavelet and $a_i,\ i = 1, 2...$, are the wavelet coefficients.
2. Obtain appropriate v order of $Q(t)$ by integration of $Q^{(n)}(t)$ with respect to t from 0 to t.
3. Substitute value of $Q^{(n)}(t)$ and all the values of $Q^{(v)}(t)$ into the given differential equation.
4. Calculate the wavelet coefficients a_i.
5. Obtain the numerical solution for $Q(t)$.

5. Computing $p_{i.v}(t)$

By Hsiao-Chen [1] method we know that

$$p_{1,i}(t) = \int_0^t h_i(t)\,dt, \tag{5.1}$$

$$p_{v,i}(t) = \int_0^t p_{v-1,i}(t)\,dt,\ v = 2, 3, ... \tag{5.2}$$

Carrying out these integrations with the aid equation (2.1), we have

$$p_{1,i}(t) = \begin{cases} t - \eta_1 & \text{for } t \in [\eta_1, \eta_2), \\ \eta_3 - t & \text{for } t \in [\eta_2, \eta_3), \\ 0 & \text{elsewhere}, \end{cases}$$

$$p_{2,i}(t) = \begin{cases} \frac{1}{2}(t-\eta_1)^2 & \text{for } t \in [\eta_1, \eta_2), \\ \frac{1}{4m^2} - \frac{1}{2}(\eta_3 - t)^2 & \text{for } t \in [\eta_2, \eta_3), \\ \frac{1}{4m^2} & \text{for } t \in [\eta_3,\ 1), \\ 0 & \text{elsewhere}. \end{cases}$$

Similarly, we can find other values of p.

6. Solution of Series L–C–R equation using Haar wavelet

The calculation is done by using 3 levels of Haar wavelet.

If an alternating E.M.F. $E\sin(wt)$ is applied to an inductance L and a capacitance C in series, then the differential equation will be (where $R = 0$)

$$L\frac{d^2Q}{dt^2} + \frac{Q}{C} = E\sin wt \qquad Q(0) = 0, \qquad Q^{'}(0) = 0. \tag{6.1}$$

Now, the complete solution of differential equation (6.1) is

$$Q(t) = C_1\cos\left(\frac{t}{\sqrt{LC}}\right) + C_2\sin\left(\frac{t}{\sqrt{LC}}\right) + \frac{EC}{(1 - LCw^2)}\ \sin(wt)$$

Also at $Q(0) = 0$ and $Q^{'}(0) = 0$ we have,

$$\begin{aligned} C_1 &= 0 \\ C_2 &= -\frac{ECw}{(1 - LCw^2)} \end{aligned}$$

Therefore, the exact solution of equation (6.1) is

$$Q(t) = -\frac{ECw}{(1 - LCw^2)}\ \sqrt{LC}\sin\left(\frac{t}{\sqrt{LC}}\right) + \ \frac{EC}{(1 - LCw^2)}\sin(wt) \tag{6.2}$$

Haar solution

Let

$$Q^{''}(t) = \sum_{i=1}^{M} a_i h_i(t) \tag{6.3}$$

Now integrating above equation with respect to t from 0 to t then we have

$$Q'(t) = \sum_{i=1}^{M} a_i p_{1,i}(t) + Q'(0)$$

Again integrating above equation with respect to t from 0 to t then we get

$$Q(t) = \left[\sum_{i=1}^{M} a_i p_{2,i}(t)\right] + \ tQ'(0) + \ Q(0)$$

After using boundary condition we get following result

$$Q(t) = \left[\sum_{i=1}^{M} a_i p_{2,i}(t)\right] \tag{6.4}$$

Now, from equation (6.1)

$$L\sum_{i=1}^{M} a_i h_i(t) + \frac{1}{C}\left[\sum_{i=1}^{M} a_i p_{2,i}(t)\right] = E\sin(wt)$$

$$\sum_{i=1}^{M} a_i \left[h_i\,(t) + \frac{p_{2,i}(t)}{LC}\right] = \frac{\mathrm{E}\sin(\mathrm{wt})}{L} \tag{6.5}$$

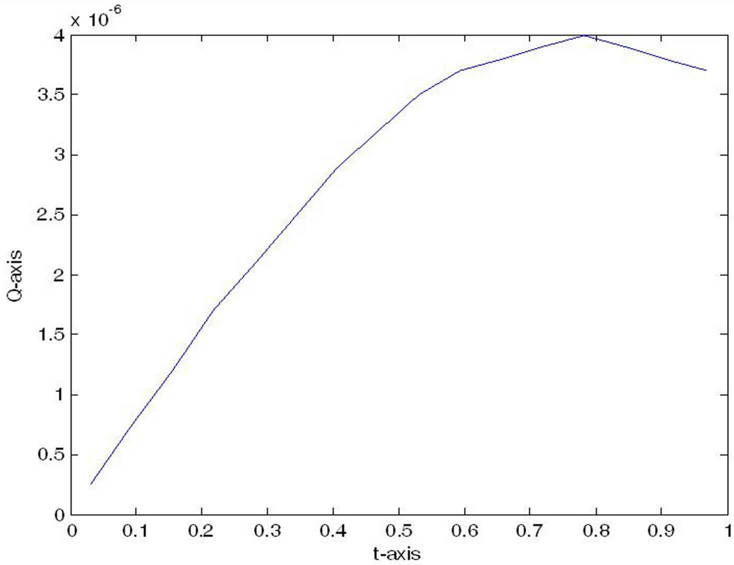
After solving system of linear differential equation we get wavelet coefficients a_i.

Case I. If $E = 2v, w = 2$ radian, $C = 2\mu F$, and $L = 1\,H$. Then equation (6.5) becomes

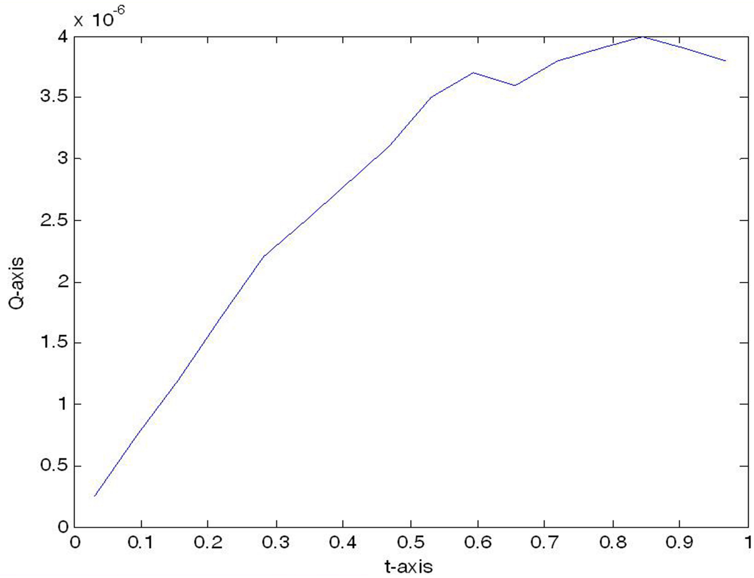
$$\sum_{i=1}^{M} a_i \left[h_i\,(t) + 10^6\frac{p_{2,i}(t)}{2}\right] = 2\sin(2t).$$

Table I
Numerical solution of Case I

t /32	Haar solution	Exact solution
1	0.00000025	0.00000025
3	0.00000074	0.00000074
5	0.0000012	0.0000012
7	0.0000017	0.0000017
9	0.0000022	0.0000021
11	0.0000025	0.0000025
13	0.0000028	0.0000029
15	0.0000031	0.0000032
17	0.0000035	0.0000035
19	0.0000037	0.0000037
21	0.0000036	0.0000038
23	0.0000038	0.0000039
25	0.0000039	0.0000040
27	0.0000040	0.0000039
29	0.0000039	0.0000038
31	0.0000038	0.0000037



Exact Figure



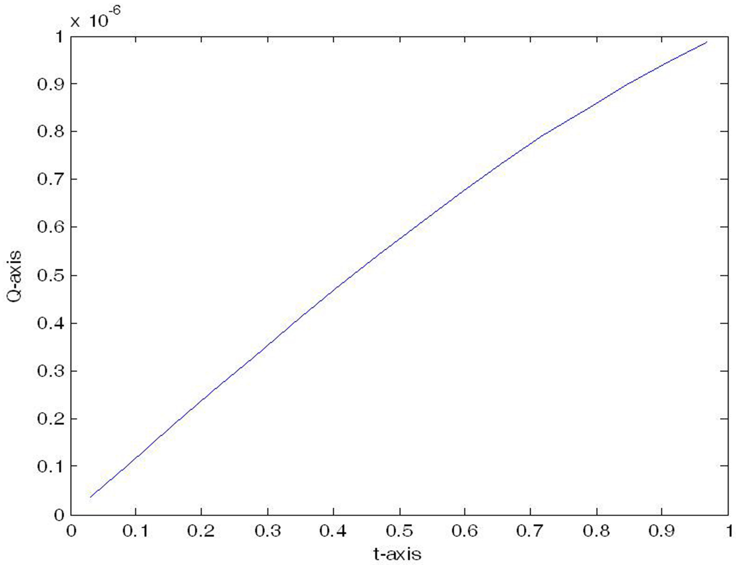
Haar Figure

Case II. If $E = 4v, w = 1$ radian, $C = 0.3\mu F$, and $L = 2H$. Then equation (6.5) becomes

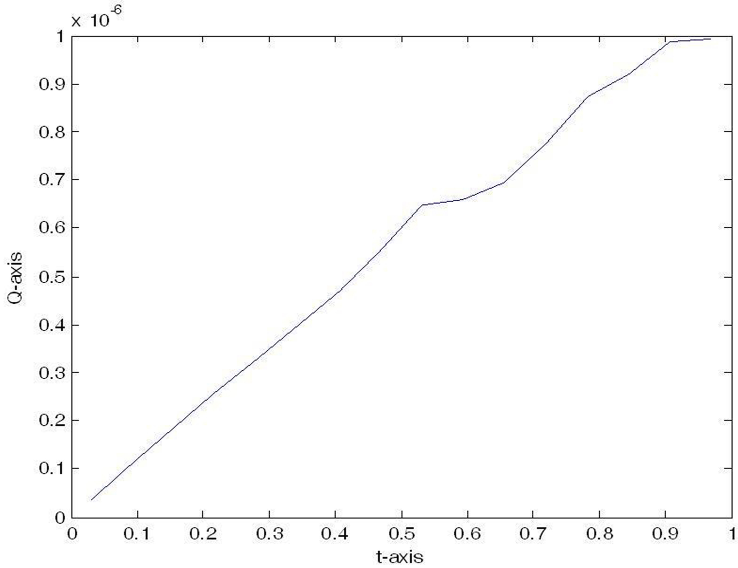
$$\sum_{i=1}^{M} a_i \left[h_i\left(t\right) + 10^6 \frac{p_{2,i}(t)}{0.6} \right] = 2\sin(t).$$

Table II
Numerical solution of Case II

t /32	**Haar solution**	**Exact solution**
1	0.000000036	0.000000037
3	0.00000011	0.00000011
5	0.00000018	0.00000018
7	0.00000026	0.00000026
9	0.00000033	0.00000033
11	0.00000039	0.00000040
13	0.00000047	0.00000047
15	0.00000055	0.00000054
17	0.00000064	0.00000060
19	0.00000065	0.00000067
21	0.00000069	0.00000073
23	0.00000077	0.00000079
25	0.00000087	0.00000084
27	0.00000091	0.00000089
29	0.00000098	0.00000094
31	0.00000099	0.00000098



Exact Figure



Haar Figure

7. Conclusion

The main goal of this paper is to demonstrate the Haar wavelet method is a powerful tool for solving linear ordinary differential equation by taking series $L-C-R$ equation. The algorithm and procedure have been applied by using Haar wavelet method for solving ODE's. The result is compared with the exact solution. It is worth mentioning that Haar solution provides excellent result even for small values of $M(M = 16)$. For large values of $M(M = 32, M = 64)$, we can also obtain the results closer to exact values.

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