H_v-STRUCTURES AND THE BAR IN QUESTIONNAIRES

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Abstract. The class of hyperstructures called H_v-structures has been studied from several aspects as well as in connection with many other topics of mathematics. Here we present applications obtained from social sciences mainly the ones used questionnaires. Moreover we improve the procedure of the filling the questionnaires, using the bar instead of Likert scale, on computers where we write down automatically the results so they are ready for research.

Key Words and Phrases: hyperstructures, H_v-structures, hopes.

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1. Basic definitions

We deal with the theory of hyperstructures introduced by Marty in 1934 [12]. For basic definitions and applications on the related theory one can see the books [3],[4],[7],[16] and related survey papers as the [6]. More specifically we focus on the large class of hyperstructures called H_v*-structures* introduced in 1990 [15], which satisfy the *weak axioms* where the non-empty intersection replaces the equality. Basic definitions on the topic are the following:

In a set H equipped with a hyperoperation (abbreviation *hyperoperation* = **hope**) $\cdot : H \times H \to P(H) - \{\varnothing\}$, we abbreviate by

WASS the *weak associativity*: $(xy)z \cap x(yz) \neq \varnothing,\ \forall x, y, z \in H$ and by

COW the *weak commutativity*: $xy \cap yx \neq \varnothing,\ \forall x, y \in H$

The hyperstructure $(H, \cdot)$ is called H_v*-semigroup* if it is *WASS*, it is called H_v*-group* if it is reproductive H_v-semigroup, i.e., $xH = Hx = H$, $\forall x \in H$. The hyperstructure $(R, +, \cdot)$ is called H_v*-ring* if both $(+)$ and $(\cdot)$ are *WASS*, the reproduction axiom is valid of $(+)$ and $(\cdot)$ is weak distributive with respect to $(+)$:

$$x(y + z) \cap (xy + xz) \neq \varnothing,\ (x + y)z \cap (xz + yz) \neq \varnothing,\ \forall x, y, z \in R.$$

Motivations. The motivation for H_v-structures is the following [16]: We know that the quotient of a group with respect to an invariant subgroup is a group. F. Marty from 1934, states that, the quotient of a group with respect to any subgroup is a hypergroup. Finally, the quotient of a group with respect to any partition (or equivalently to any equivalence relation) is an H_v-group.

Specifying this motivation we remark: Let $(G, \cdot)$ be a group and R be an equivalence relation (or a partition) in G, then $(G/R, \cdot)$ is an H_v-group, therefore we have the quotient $(G/R, \cdot)/\beta^*$ which is a group, the *fundamental* one. Remark that the classes of the fundamental group $(G/R, \cdot)/\beta^*$ are a union of some of the R-classes. Otherwise, the $(G/R, \cdot)/\beta^*$ has elements classes of G where they form a partition which classes are larger than the classes of the original partition R.

In an H_v-semigroup the powers of an element $h \in H$ are defined as follows: $h^1 = \{h\}, h^2 = h \cdot h, ..., h^n = h \circ h \circ ... \circ h$, where $(\circ)$ denotes the *n-ary circle hope*, i.e. take the union of hyperproducts, n times, with all possible patterns of parentheses put on them. An H_v-semigroup $(H, \cdot)$ is called *cyclic of period s*, if there exists an element g, called *generator*, and a natural number s, the minimum one, such that $H = h^1 \cup h^2 ... \cup h^s$. Analogously the cyclicity for the infinite period is defined [16]. If there is an element h and a natural number s, the minimum one, such that $H = h^s$, then $(H, \cdot)$ is called *single-power cyclic of period s.*

The main tool to study hyperstructures are the fundamental relations β^*, γ^* and ϵ^*, which are defined, in H_v-groups, H_v-rings and H_v-vector spaces, resp., as the smallest equivalences so that the quotient would be group, ring and vector space, resp. The relation β^* was introduced by M. Koskas in 1970 [11] and was mainly studied intensively and in depth by Corsini [3]. The relations γ^* and ϵ^*, were introduced by T. Vougiouklis [15],[16],[17] and he named them Fundamental. A way to find the fundamental classes is given by theorems as the following [16]:

Theorem 1.1 *Let $(H, \cdot)$ be an H_v-group and denote by $\boldsymbol{U}$ the set of all finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x, y\} \subset \boldsymbol{u}$ where $u \in U$. Then β^* is the transitive closure of β.*

An element is called single if its fundamental class is singleton [16].

Fundamental relations are used for general definitions. Thus, an H_v-ring $(R, +, \cdot)$ is called H_v-field if R/γ^* is a field.

Let $(H, \cdot), (H, *)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called smaller than $(*)$, and $(*)$ greater than $(\cdot)$, iff there exists an

$$f \in Aut(H, *) \text{ such that } xy \subset f(x * y),\ \forall x, y \in H.$$

Then we write $\cdot \leq *$ and we say that $(H, *)$ contains $(H, \cdot)$. If $(H, \cdot)$ is a structure then it is called *basic structure* and $(H, *)$ is called $H_b - structure$.

Theorem 1.2 (The Little Theorem). *Greater hopes than the ones which are WASS or COW, are also WASS or COW, respectively.*

This Theorem leads to a partial order on H_v-structures and mainly to a correspondence between hyperstructures and posets. The determination of all H_v-groups and H_v-rings is very interesting but hard.

To compare classes we can see the small sets. The problem of enumeration and classification of H_v-structures, or of classes of them, was started very early but recently we have interesting results by using computers. The problem is complicate in H_v-structures because we have great numbers. The partial order introduced in H_v-structures restrict the problem in finding the minimal, up to *isomorphisms*, H_v-structures. In this direction we have recently results by Bayon and Lygeros as the following [1]:

In a set with two elements, then there are 20 H_v-groups, up to isomorphism.

In sets with three elements: There are 6.494 minimal isomorphisms-groups. The 137 are abelians and 6.357 are not; the 6.152 are cyclic and 342 are not. The number of H_v-groups with three elements is 1.026.462. The 7.926 are abelians, 1.018.536 are not; 1.013.598 are cyclic and 12.864 are not. 16 are very thin.

The number of H_v-groups with 4 elements with scalar unit is 631.609. There are 8.028.299.905 abelian H_v-groups, the 7.995.884.377 are cyclic and the 32.415.528 are not. There are 10.614.362 abelian hypergroups: the 10.607.666 are cyclic and the 6.696 are not. Notice that there are only 97 canonical hypergroups.

Definition 1.3 [18],[19]. Let $(H, \cdot)$ be hypergroupoid. We *remove* $h \in H$, if we consider the restriction of $(\cdot)$ in the set $H - \{h\}$. $\underline{h} \in H$ *absorbs* $h \in H$ if we replace h by $\underline{h}$ and h does not appear in the structure. $\underline{h} \in H$ merges with $h \in H$, if we take as product of any $x \in H$ by $\underline{h}$, the union of the results of x with both h, $\underline{h}$, and consider h and $\underline{h}$ as one class with representative $\underline{h}$, therefore, h does not appear in the hyperstructure.

In 1989 Corsini and Vougiouklis introduced a method to obtain stricter algebraic structures from given ones through hyperstructure theory. This method was introduced before of the H_v-structures, but in fact the H_v-structures appeared in the procedure.

Definition 1.4 The *uniting elements method* is the following: Let G be a structure and d be a property, which is not valid, and it is described by a set of equations. Consider the partition in G for which it is put together, in the same class, every pair of elements that causes the non-validity of d. The quotient G/d is an H_v-structure. Then quotient of G/d by the fundamental relation β^*, is a stricter structure $(G/d)\beta^*$ for which d is valid.

An application of the uniting elements is if more than one property desired. The reason for this is some of the properties lead straighter to the classes: commutativity and the reproductivity are easily applicable. One can do this because there is a related theorem [16].

The Lie-Santilli *isotopies* born to solve Hadronic Mechanics problems. Santilli proposed [13] a 'lifting' of the trivial unit matrix of a normal theory into a nowhere singular, symmetric, real-valued, new matrix. The original theory is reconstructed such as to admit the new matrix as left and right unit. The *isofields* needed correspond to H_v-structures called *e-hyperfields* which are used in physics or biology. *Definition*: Let $(H_o, +, \cdot)$ be the attached H_v-field of the H_v-semigroup $(H, \cdot)$. If $(H, \cdot)$ has a left and right scalar unit e, then $(H_o, +, \cdot)$ is an e-hyperfield, the *attached* H_v-field of $(H, \cdot)$.

Most of H_v-structures are used in Representation (abbreviate by rep) Theory. Reps of H_v-groups can be considered either by generalized permutations or by H_v-matrices [14],[16],[19]. Reps by generalized permutations can be achieved by using translations. In the rep theory the singles are playing a crucial role.

The rep problem by H_v-matrices is the following:

H_v-*matrix* is called a matrix if has entries from an H_v-ring. The hyperproduct of H_v-matrices $A = (a_{ij})$ and $B = (b_{ij})$, of type $m \times n$ and $n \times r$, respectively, is a set of $m \times r$ H_v-matrices, defined in a usual manner:

$$A \cdot B = (a_{ij}) \cdot (b_{ij}) = \{C = (c_{ij}) | c_{ij} \in \oplus \sum a_{ik} \cdot b_{kj}\},$$

where $(\oplus)$ denotes the n-ary circle hope on the hyperaddition.

Definition 1.5 Let $(H, \cdot)$ be an H_v-group, $(R, +, \cdot)$ an H_v-ring, $M_R = \{(a_{ij}) | a_{ij} \in R\}$, then any map

$$\mathbf{T} : H \to \mathbf{M}_R : h \to T(h) \textit{ with } T(h_1 h_2) \cap T(h_1)T(h_2) \neq \varnothing, \forall h_1, h_2 \in H,$$

is called H_v-*matrix rep*. If $T(h_1h_2) \subset T(h_1)T(h_2)$, then $\boldsymbol{T}$ is an *inclusion rep*, if $T(h_1h_2) = T(h_1)T(h_2)$, then $\boldsymbol{T}$ is a *good rep*.

Hopes on any type of matrices can be defined, these are called helix hopes [8], [25].

2. The ∂-hopes

In [20],[21],[22] we defined a hope, in a groupoid with a map f on it called *theta* ∂.

Definition 2.1 Let $(G, \cdot)$ be groupoid (resp., hypergroupoid) and $f : G \to G$ be a map. We define a hope (∂), on G as follows

$x \partial y = \{f(x) \cdot y, x \cdot f(y)\}$, $\forall x, y \in G$. (resp. $x \partial y = (f(x) \cdot y) \cup (x \cdot f(y))$, $\forall x, y \in G$

If $(\cdot)$ is commutative then (∂) is commutative. If $(\cdot)$ is COW then (∂) is COW.

Let $(G, \cdot)$ be groupoid (resp., hypergroupoid) and $f : G \to P(G) - \{\varnothing\}$ be any multivaued map. We define the (∂), on G as follows $x \partial y = (f(x) \cdot y) \cup (x \cdot f(y))$, $\forall x, y \in G$.

Let $(G, \cdot)$ be groupoid $f_i : G \to G, i \in I$, be a set of maps on G. The $f_\cup : G \to P(G) : f_\cup(x) = \{f_i(x) | i \in I\}$, is the *union* of $f_i(x)$. We have the *union theta-hope* (∂), on G if we take $f_\cup(x)$. If we take $\underline{f} \equiv f \cup (id)$, then we have the *b-theta-hope*.

Motivation for the definition of the theta-hope is the map *derivative* where only the multiplication of functions can be used.

Properties 2.2 *If $(G, \cdot)$ is a semigroup, then:*

1. *For every f, the (∂) is WASS*
2. *If f is homomorphism and projection , i.e $f^2 = f$, then (∂) is associative.*
3. *If $(G, \cdot)$ is a semigroup then, for every f, the b-theta-hope (∂) is WASS.*
4. *Reproductivity. If $(\cdot)$ is reproductive then (∂) is also reproductive.*
5. *Commutativity. If $(\cdot)$ is commutative then (∂) is commutative. If f is into the centre of G, then (∂) is a commutative. If $(\cdot)$ is a COW then, (∂) is a COW.*
6. *Unit elements. u is right unit if $x\partial u = \{f(x)\cdot u, x\cdot f(u)\} \ni x$. So$f(u) = e$, if e is a unit in $(G, \cdot)$.The elements of the kernel of f, are the units of (G, ∂). In hypergroups does not necessarily exist any unit element and if there exists a unit this is not necessarily unique. Moreover the ∂-hopes do not have always the unit element of the group as unit for the corresponding ∂-hope. This is so because $e\partial e = \{f(e)e, ef(e)\} = \{f(e)\}$*
7. *Inverse elements. Let $(G, \cdot)$ be a monoid with unit e and u be a unit in (G, ∂), then $f(u) = e$.. For given x, the x' is an inverse with respect to u, if $x\partial x' = \{f(x) \cdot x', x \cdot f(x')\} \ni u$ and $x'\partial x = \{f(x') \cdot x, x' \cdot f(x)\} \ni u$. So, $x' = (f(x))^{-1}u$ and $x' = u(f(x))^{-1}$ are the right and left inverses, respectively.We have two-sided inverses iff $f(x)u = uf(x)$*

Proposition 2.3 *Let $(G, \cdot)$ be, then for all maps $f : G \to G$, the (G, ∂) is an H_v-group.*

Motivation. For the definition of the theta-hope is the map *derivative* where only the multiplication of functions can be used. Therefore, in these terms, for two functions $s(x)$, $t(x)$, we have $s\partial t = \{s't, st'\}$, where $(')$ denotes the derivative.

Proposition 2.4 *Let $g \in G$ is a generator of the group $(G, \cdot)$. Then,*

(a) *for every f, g is a generator in $(G\partial)$, with period at most n.*

(b) *suppose that there exists an element w such that $f(w) = g$, then the element w is a generator in (G, ∂), with period at most n.*

There is connection of ∂-hopes with other hyperstructures:

Example. P-hopes [16]. Let $(G, \cdot)$ be a commutative semigroup and $P \subset G$. Consider the multivalued map f such that $f(x) = P \cdot x,\ \forall x, y \in G$.

Then we have $x\partial y = x \cdot y \cdot P,\ \forall x, y \in G$.

So, the ∂-hope coincides with the well known class of P-hopes [22].

One can define ∂-hopes on rings and other more complicate structures, where more than one ∂-hopes can be defined. Moreover, one can replace structures by hyper ones or by H_v-structures, as well.

3. The bar in questionnaires

During last decades hyperstructures seem to have a variety of applications not only in other branches of mathematics but also in many other sciences including the social ones. These applications range from biomathematics and hadronic physics to automata theory, to mention but a few. This theory is closely related to fuzzy theory; consequently, hyperstructures can now be widely applicable in industry and production, too.

In several papers, such as [2], [5], [13], [24], one can find numerous applications; similarly, in the books [4], [7] a wide variety of applications is also presented.

An important new application, which combines hyperstructure theory and fuzzy theory, is to replace in questionnaires the scale of Likert by the bar of Vougiouklis & Vougiouklis. The suggestion is the following [10]:

Definition 3.1 "In every question substitute the Likert scale with 'the bar' whose poles are defined with '0' on the left end, and '1' on the right end:

$$0 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 1$$

The subjects/participants are asked instead of deciding and checking a specific grade on the scale, to cut the bar at any point s/he feels expresses her/his answer to the specific question".

The use of the bar of Vougiouklis & Vougiouklis instead of a scale of Likert has several advantages during both the filling-in and the research processing. The final suggested length of the bar, according to the Golden Ratio, is 6.2cm, see [24]. Several advantages on the use of the bar instead of scale one can find in [10].

4. A computerizing filling questionnaires

We present now a program of filling a questionnaire on a computer such that the results automatically can be transferred for research elaboration.

There are several advantages of the bar one of them is the time of filling the questionnaire. The only disadvantage of the bar is to transfer the data collection to a computer for elaboration. At this point, we present an implemented application to overcome the problems raised during the transferring the data. This application overcomes the problem of inputting data from questionnaires to processing and eliminates time of data collection, transferring data directly for any kind of elaboration.

The application has been implemented using Visual Basic and the data is being saved on a Microsoft Access Database. The application is based on "events" and an OleDbConnection is used to connect the program with the database.

Filling-in such questionnaire can be easily achieved by using this application, as it is based on a very simple user interface. The participants have to "click" on the bar, in order to indicate the point that satisfies their answer on the question made. The user has the opportunity to change his answer by "clicking" on another point anytime before submit.

The results are being saved on a simple database (Microsoft Access Database) indicating the exact point each participant has "cut" the bar

5. Applications

One problem in research is to describe mathematical models using theta-hopes. Such a problem is the following [23]:

Problem 5.1 *In the research processing suppose that we want to use Likert scale dividing the continuum* [01] *both by, first, into equal steps (segments) and, second, into equal-area spaces according to Gauss distribution* [9], [24]. *If we consider both types of divisions into n segments, then the continuum* [01] *is divided into* $2n-1$ *segments, if n is odd number and into* $2(n-1)$ *segments, if n is even number. We can number the segments and we can consider as an organized devise the group* $(Zk, \oplus)$ *where* $k = 2n-1$ *or* $2(n-1)$. *Then we can obtain several hyperstructures using* ∂*-hopes as the following way: We can have two partitions of the final segments, into n classes either using the division into equal steps or the Gauss distribution by putting in the same class all segments that belong* (a) *to the equal step or* (b) *to equal-area spaces according to Gauss distribution. Then we can consider two kinds of maps* (i) *a multi-map where every element corresponds to the hole class or* (ii) *a map where every element corresponds to one special fixed element of the same class. Using these maps we define the* ∂*-hopes and we obtain the corresponding* H_v*-structure.*

An application on this direction is the following construction [23]:

Construction 5.2 *Consider a group* $(G, \cdot)$ *and suppose take a partition* G_i, $i \in I$, *of the* ***G***. *Select and fix an element* g_i *of each partition class* G_i, *and consider the map*

$$f : G \to G \text{ such that } f(x) = g_i, \ \forall x \in G_i,$$

then (G, ∂) *is an* H_v*-group. Moreover, the fundamental group* $(G/R, \cdot)/\beta^*$ *is* (*up to isomorphism*) *a subgroup of the corresponding fundamental group* $(G, \partial)/\beta^*$.

Remark. In the above construction, if one of the selected elements is the unit element e of the group $(G, \cdot)$, otherwise, if there exist an element $z \in G$ such that $f(z) = e$, then we have $(G/R, \cdot)/\beta^* = (G, \partial)/\beta^*$.

Proposition 5.3 *Suppose* $(G, \cdot)$ *be a group and* G_i, $i \in I$ *be a partition of* G. *For any class we fix a* $g_i \in G_i$, *and take the map* $f : G \to G : f(x) = g_i, \forall x \in G_i$. *If for the unit element e, in* $(G, \cdot)$, *we have* $f(e) = e$, *i.e. e is any fixed element, then e is also a unit element of the* H_v*-group* (G, ∂). *Moreover* $(f(x))^{-1}$ *is an inverse element in the* ∂*-*H_v*-group* (G, ∂), *of x.*

Now, we conclude with an example of the above Construction:

Example 5.4 Suppose that we take the case of the Likert scale with 5 equal steps: $[0 - 1.24 - 2.48 - 3.72 - 4.96 - 6.2]$ and the Gauss 5 equal areas: $[0 - 2.4 - 2.9 - 3.3 - 3.8 - 6.2]$ we have 9 segments as follows

$$[0 - 1.24 - 2.4 - 2.48 - 2.9 - 3.3 - 3.72 - 3.8 - 4.96 - 6.2]$$

Therefore, if we consider the set $(Z_9,+)$ and if we name the above segments by 0, 1, 2, ..., 8 then if we consider the Gauss partition: $\{0,1\}, \{2,3\}, \{4\}, \{5,6\}, \{7,8\}$ we take, according to the above Construction, the map f such that $f(0) = 0$, $f(1) = 0$, $f(2) = 2$, $f(3) = 2$, $f(4) = 4$, $f(5) = 5$, $f(6) = 5$, $f(7) = 7$, $f(8) = 7$, then we obtain the following table:

∂	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	0	0,1	2	2,3	4	5	5,6	7	7,8
1	0,1	1	2,3	3	4,5	5,6	6	7,8	8
2	2	2,3	4	4,5	6	7	7,8	0	0,1
3	2,3	3	4,5	5	6,7	7,8	8	0,1	1
4	4	4,5	6	6,7	8	0	0,1	2	2,3
5	5	5,6	7	7,8	0	1	1,2	3	4,3
6	5,6	6	7,8	8	0,1	1,2	2	3,4	4
7	7	7,8	0	0,1	2	3	3,4	5	5,6
8	7,8	8	0,1	1	2,3	4,3	4	5,6	6

Remark that, for the H_v-group (Z_9, ∂), the elements 0 and 1 are unit elements. (Z_9, ∂) is cyclic where the elements 2, 3, 4, 5, 6, 7 and 8 are generators with period 6, 7, 6, 9, 6, 7 and 7 respectively.

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