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Continuity in mathematics learning beyond the summer: classroom, outdoor environments, and the design of resilient mathematical experiences

1. Introduction: continuity in mathematics learning beyond classroom boundaries

Continuity is a familiar concern in mathematics education. It is often discussed in relation to curriculum progression, transitions between school levels, or the coherence of mathematical contents over time. The centrality of this construct is discussed in what follows. Students do need opportunities to revisit, deepen, and reorganise mathematical ideas across grades and institutional transitions, and in any occasion of possible leap within their learning trajectory. Yet the contributions collected in this special issue invite us to broaden the meaning of continuity. Continuity in mathematics learning is not only a matter of what comes before and after in a curriculum. It also concerns the possibility for mathematical experiences, practices, meanings, and forms of participation to remain available when the conditions of schooling change.

The contributions collected address this matter through a range of situated cases: a summer web app for children and families, a kindergarten game re-enacted at home, outdoor explorations of shadows and math trails, museum-based work on spirals and mathematical machines, online and AI-mediated feedback and teacher education initiatives focused on intercultural and formative mediation. Across these cases, continuity is not treated as the mere persistence of content, but as the possibility of reactivating mathematical work through different mediations, settings, and forms of participation.

The title of this special issue, *Continuity and learning beyond the summer. Mathematics between classroom and outdoor environments: research perspectives in dialogue*, captures this double movement. On the one hand, the summer break foregrounds the temporal dimension of continuity: what happens to mathematical activity when the ordinary rhythm of schooling is interrupted? On the other hand, the reference to classroom and outdoor environments foregrounds the spatial and cultural dimension of continuity: how does mathematical activity change when it moves across places, artefacts, social relations, institutional aims, and forms of mediation?

The issue originates as a follow-up to the conference held at the University of Salerno on 19 February 2026, within the PRIN2022 project *Coming to terms not only with the pandemic. Mathematics learning loss in primary school: underlying factors and interventions* (Prot. 2022TWCJAS, CUP D53D23013070006). However, it is not limited to the results of that project, nor to the papers presented at the conference. Its aim is broader: to offer a composite view of research, design experiences, and teacher education initiatives concerned with the continuity of mathematics learning across classroom, outdoor, home, digital, museum-based, intercultural, or other non-strictly formal educational settings.

The special issue is therefore framed by a shift of perspective: from summer learning loss understood as decline to continuity understood as the design of sustained opportunities for mathematical activity during the summer; from the classroom as the only legitimate site of mathematical work to a network of settings that includes homes, schoolyards, museums, digital environments and teacher education workshops; from isolated activities to trajectories in which mathematical meanings are prepared, transformed, documented, and taken up again.

2. From learning loss to learning continuity

The PRIN2022 framework places the special issue in a post-pandemic research landscape, but the problem addressed in this Special Issue, four years later, is broader than pandemic-related interruption. The pandemic made visible forms of discontinuity that were already present in educational systems: unequal access to learning opportunities, fragile connections between school and family contexts, and difficulties in sustaining mathematical engagement when ordinary classroom routines are interrupted. The summer break provides a recurrent interruption in school time through which researchers can examine how mathematical engagement changes when regular classroom routines, teacher mediation, and shared school rhythms are temporarily suspended.

In mathematics, this issue is particularly delicate because mathematical learning depends not only on remembering procedures, but also on maintaining opportunities for reasoning, representing, arguing, modelling, and problem solving over time. The contributions connected to the MaTEs project address summer continuity from different angles: the design and use of a web app for children and families, the analysis of young pupils' probabilistic representations, and the study of parental stereotypes in early mathematical attitudes. In these studies, summer learning loss is treated not only as the possible decline of procedural fluency, but as the fragility of sustained engagement with non-routine problems, narrative modelling, drawings, justification, and adult-mediated mathematical discussion during periods in which school routines are reduced or absent.

The contribution by Albano et al. addresses this issue directly through the MaTEs web app, designed to support children and families during the summer period. The study does not treat the app simply as a device for assigning additional exercises. Rather, it frames it as a hybrid environment in which digital prompts, offline mathematical work, adult mediation, narrative contexts, documentation of children's productions, and affective-metacognitive reflection are integrated. The app is explicitly not conceived as an automated learning environment, but as a mediating device between the child, the adult, and the mathematical activity.

The empirical analysis of log data from 315 users does not claim to measure learning outcomes directly. It examines usage trajectories during the summer, identifying different patterns in terms of completion, continuity, and temporal distribution. This is important because it prevents a simplistic relation between access to a resource and educational continuity. The findings show that use was often non-uniform and sometimes concentrated toward the end of the summer, while the largest group displayed a more regular and sustained pattern. The result gives empirical substance to one of the issue's central claims: continuity cannot be inferred from the provision of a tool alone. It has to be examined through temporal patterns of use, the distribution of activity across the summer, and the forms of adult-mediated engagement that the resource makes possible.

At the same time, the issue invites caution toward a narrow deficit-oriented understanding of learning loss. Bianco et al. enrich the notion of loss by relocating it within multicultural classrooms. Their argument is that, in some cases, mathematical knowledge is not lost but becomes institutionally unrecognisable when students' algorithms, languages, symbolic systems, or epistemological habits do not coincide with those expected by the dominant school culture.

In this perspective, what is often interpreted as missing knowledge may instead be knowledge that has been set aside because it is not readily interpretable within the host classroom.

This reframing is crucial for the special issue. It prevents learning loss from becoming a purely compensatory category. The issue is not only how to recover what may have been lost, but how to recognise what has remained invisible, how to sustain what risks being interrupted, and how to design mathematical experiences that can remain meaningful across changes in time, place, language, and social expectation. The response proposed by Bianco, Di Paola and Nicosia is not remediation in the narrow sense, but teacher education aimed at preparing teachers to act as cultural brokers: professionals able to recognise, interpret, and value mathematical knowledge even when it is produced in distant or unfamiliar contexts.

In this sense, the movement from learning loss to learning continuity is not merely terminological. This does not mean dismissing the empirical problem of learning loss, nor the need to document changes in students' mathematical competence across interruptions. Rather, it means placing such documentation within a broader design question: which forms of mathematical activity remain available to students when school time is suspended, and which forms require intentional mediation in order not to disappear?

2.1. Mathematics between classroom, outdoor environments, and other educational settings

Outdoor mathematics education occupies a central position in this special issue, but the contributions make clear that outdoor learning should not be understood as the simple relocation of classroom tasks to an open-air setting. In the papers focused on outdoor or place-based settings, the educational relevance of the environment does not derive simply from being outside the classroom. It emerges when the environment becomes part of the mathematical task: shadows cast by gnomons in the schoolyard, measurements and observations required by a math trail, spirals encountered through museum objects and bodily movement.

De Giorgi's contribution develops this point through the field of experience of sun and shadows. The paper connects Outdoor Education, fields of experience, instrumental orchestration, semiotic mediation, and teacher professional development. The analysed episode in a fourth-grade classroom and schoolyard shows that the outdoor setting is not an accessory to learning. It becomes part of the mathematical work itself. Students explore shadows through gnomons, drafting triangles, bodily movement, and observations of objects in the courtyard; the teacher orchestrates these explorations by drawing attention to relationships such as direction, parallelism, shadow length, and the relation between light source and object.

The provisional notion of *Outfield Education* is useful here because it avoids a romanticised view of outdoor work. It describes an education that takes place in, about, and through the field of experience, where the field itself functions as a learning environment connecting formal inquiry, outdoor exploration, digital simulations, and students' lived worlds. At the same time, De Giorgi's analysis makes visible the fragility of such work. Teachers' reflections point to the need for further teacher education, more confidence in managing outdoor mathematical activities, and stronger curricular integration, possibly through multi-year pathways.

Taranto and Distefano similarly show that outdoor mathematics depends on design, mediation, and reflection. Their study of a non-digital math trail in a fifth-grade class demonstrates that some core features of digital math trails can be preserved when smartphones are unavailable or restricted. Treasure-hunt activities support spatial orientation; paper envelopes with progressive hints scaffold problem solving; cooperative roles organise participation. Yet the analog redesign does not fully replace the affordances of digital tools. The absence of immediate digital feedback makes validation and assessment more demanding, shifting greater responsibility to classroom discussion and to the teacher's reconstruction of students' written and oral processes.

The study also warns against assuming that cooperation automatically supports all phases of mathematical modelling. While group work can foster engagement and shared reasoning, interpretation and validation still require explicit scaffolding and opportunities for collective discussion.

Casi's theoretical-methodological paper extends this reflection to classroom–museum continuity. It does not treat museums as outdoor settings in a classificatory sense. Rather, it uses an outdoor lens analytically, to make visible place, materiality, movement, mediation, and contextual difference in classroom–museum trajectories. The point is not that the museum is outside school in a simple spatial sense, but that it configures mathematical work through objects, rhythms, institutional purposes, and forms of participation that differ from classroom practice.

The paper argues that continuity cannot be reduced to the recurrence of a mathematical topic across classroom and museum. In the worked example, students move between classroom explorations of circumferences and a museum workshop on spirals, where distance from a centre, rotation, variation of distance, bodily movement, architectural details, and mathematical machines such as the spiralograph and the helicograph become part of the same broader trajectory. What travels across contexts is not only a topic, but a set of ways of seeing, moving, describing, conjecturing, and using artefacts mathematically.

Together, these contributions suggest that the classroom/outdoor relation should not be conceived as a binary opposition. The classroom remains essential as a place for preparation, discussion, institutionalisation, comparison, and reflection. Outdoor environments, museums, schoolyards, and local territories offer forms of experience that cannot be fully reproduced in the classroom. Continuity emerges when these settings are not treated as isolated episodes, but as parts of trajectories in which mathematical meanings can be prepared, transformed, documented, and reactivated.

2.2. Mediation across home, family beliefs, and cultural recognition

Across the issue, the relation between school and what lies beyond it appears in several different forms. In Soldano and Casi, it is a home-school relationship mediated by children who re-enact a mathematical game with parents. In Uberti, it is a family belief system that shapes children's early mathematical attitudes and gendered self-perceptions. In Bianco et al. it is a cultural relation between students' mathematical backgrounds and the dominant norms of the classroom. These are not the same phenomenon, but they converge on one point: continuity depends on whether mathematical activity is recognised as meaningful by those who participate in it.

Soldano and Casi's study on *Betta-the-Bee* offers a particularly clear example. A mathematically rich card game is introduced at kindergarten and then taken home by children during the Christmas break. The game itself is structured around the formulation of yes/no questions, the recognition of variables, the interpretation of answers, and the elimination of possibilities. Its mathematical specificity therefore lies not only in the visible features of the deck, but in the rule-governed reasoning enacted during play.

The findings show that children do not simply transport a material object. They mediate a school-shaped mathematical practice. They introduce the game, explain the rules, correct adult misunderstandings, and keep the interaction aligned with what counts as proper play. The continuity observed in the study depends on more than the portability of the game: what children carry home is a rule-governed practice already socialised at school.

This has consequences for parental recognition of mathematics. After the home experience, parents no longer refer mainly to visible numerical content. They more often identify logical thinking, grouping, spatial organisation, and the role of questioning. The window of what becomes noticeable as mathematics appears to widen. Continuity here is therefore not only the reuse of the same game across settings, but the transformation of what parents are able to recognise as mathematical within that activity.

Uberti's analysis of parental stereotypes and early gender gaps in mathematical attitudes adds another layer. Drawing on data from the MATES project, the study shows that gender differences in mathematical attitudes are already observable among 8-year-old children: boys display slightly higher self-efficacy and stronger liking for mathematics, while girls report higher overall school enjoyment. No significant differences emerge in the use dimension. The parental dimension is particularly important: stereotypes such as the belief that males are more naturally suited to mathematics are associated with lower liking for mathematics and self-efficacy among girls, while reinforcing more positive attitudes among boys.

This contribution shows that continuity is shaped not only by tasks and resources, but also by the symbolic environment in which children learn to see themselves as mathematical subjects. Family beliefs do not simply accompany learning; they contribute to the formation of expectations, self-efficacy, and early perceptions of who can legitimately see themselves as mathematically competent.

Bianco et al. broaden the problem further by shifting the focus from family beliefs to cultural recognition in multicultural classrooms. Survey data from 560 in-service teachers reveal that only a small proportion had received training specifically focused on mathematics teaching in plurilingual and multicultural contexts. Teachers identify language barriers as the most significant difficulty, but also mention cultural differences, didactic issues, institutional constraints, and lack of family involvement.

The authors' response is grounded in teacher education and co-design. The shared planning scheme developed within the "Cultures Count" course aims to help teachers attend to class composition, students' interests and habits, prior knowledge, linguistic and cultural differences, and the relation between institutional objectives and students' lived mathematical experiences. The teacher is positioned not merely as a transmitter of content, but as a cultural mediator and broker of meanings. Continuity here means reducing the distance between classroom and home cultures, between dominant and subaltern mathematical practices, and between institutional expectations and students' previous knowledge.

Taken together, these papers suggest that continuity cannot be designed only at the level of tasks or materials. It also depends on mechanisms of recognition: whether parents come to see questioning and elimination as mathematical in a game; whether girls and boys receive different implicit messages about who is "naturally" suited to mathematics; whether teachers can recognise culturally different algorithms and mathematical practices as resources rather than deficits.

2.3. Storytelling, representation, affect, and feedback

A further cross-cutting theme concerns the affective, narrative, representational, and feedback-mediated conditions of continuity. Students do not sustain mathematical engagement only because tasks are cognitively meaningful. They also need reasons to participate, emotional conditions that make persistence possible, representations that allow them to organise thinking, and feedback that supports revision without reducing mathematics to correctness alone.

Coppola and Sassone's contribution on *Mortino and the Blue Pearl Heart* explores this issue through a structured narrative and gamified pathway in a second-grade classroom. The study does not present storytelling and gamification as devices that simply remove anxiety. Rather, it shows how a sustained narrative trajectory can provide a shared context in which pupils externalise uncertainty, persist in demanding tasks, and begin to experience mathematical problems as meaningful challenges within a storyworld. The design is layered: the narrative forms the outer shell, while gamification constitutes an intermediate level, and mathematics remains the core of the experience. Identification with Mortino supports children's sense of responsibility and participation, while the mathematical challenges allow the story to progress.

This contribution is important for the special issue because it frames continuity not only as temporal persistence, but also as narrative and affective coherence. Mathematical activity is embedded in a layered design in which the narrative provides continuity over time, gamification sustains participation, and mathematical tasks remain the core through which the story progresses.

The paper by Andrà et al. on probability and drawings foregrounds representation as another condition for continuity, especially within the MaTEs concern for designing meaningful mathematical tasks for young children beyond routine school exercises. By asking Grade 2 pupils to draw and justify their answers in a probabilistic task, the study shows how drawings may function as semiotic resources through which children coordinate intuition, narrative elements, and mathematical structure. The authors deliberately move beyond a view of probabilistic thinking centred only on misconceptions and instead attend to the intertwined nature of intuitions, procedures, and representations. Their analysis shows that even at this young age, children can identify and employ mathematical features of a probabilistic task in order to answer it correctly, while many still struggle to provide written justifications for correct choices.

This result is relevant beyond probability. It suggests that continuity between intuition, representation, and formal reasoning cannot be assumed. It must be supported through semiotic resources that allow children to move from experience to mathematical articulation. Drawings function as intermediate semiotic resources: they allow pupils to display aspects of the probabilistic situation that may not yet be articulated in written justification.

Two contributions address feedback and AI, extending the issue's concern with continuity to digital and feedback-mediated environments. Vitale et al. examine informal online help-seeking in formal logic, where mathematical support is asynchronous, voluntary, and affectively fragile. Their study compares human feedback with LLM-generated feedback, evaluating empathy through emotion recognition, perspective-taking, and emotional transmission. The findings indicate that LLM-generated feedback, when supported by structured prompts, is rated as more empathetic than human feedback, especially in emotion recognition and emotional transmission, while perspective-taking remains the most complex dimension.

Fiorentino et al. examine a different setting: pre-service teacher education. Their study interprets an Artificial Agent developed through ChatGPT 5.2 as a possible dynamic semiotic mediator within formative feedback processes. The activity involved 300 pre-service teachers working in pairs on an open-ended mathematical problem, followed by interaction with the Artificial Agent and collective discussion. The qualitative analysis of 150 protocols shows a tendency to move from local, incomplete, or poorly argued responses toward broader, more structured, and more generalised responses. At the same time, the authors identify critical issues: misleading feedback, overly verbose outputs, and the risk of mechanical use. These become formative only when discussed and mediated by the teacher.

The two AI-related contributions extend the issue's concern with continuity in two distinct directions. Vitale et al. examine continuity as affective and cognitive support in informal digital help-seeking, where feedback may sustain or discourage learners' willingness to continue working on mathematical problems. Fiorentino et al. examine continuity as revision within a designed formative sequence, where AI-generated feedback becomes productive only through prompt design, peer interaction, and collective teacher-mediated discussion. In both cases, the central issue is not whether AI can replace human mediation. The issue is how feedback, human or artificial, can become part of a designed trajectory of revision, reflection, and mathematical meaning-making.

3. The rationale and scope of the special issue

The contributions collected in this issue do not form a homogeneous set in terms of methods, levels, or theoretical frameworks. They include empirical case studies, quantitative analyses, theoretical-methodological proposals, teacher education studies and digital learning studies. This plurality is not a weakness. It is precisely what allows the special issue to address continuity in its complexity.

Across the papers, continuity appears in several forms. It is temporal, when mathematical engagement is sustained, interrupted, postponed, or intensified during the summer break. It is spatial, when mathematical work moves between classroom, schoolyard, museum, home and digital environments. It is cultural, when learning depends on whether students' prior knowledge, family beliefs, language, gendered expectations, and cultural practices are recognised. It is affective, when students' willingness to engage depends on confidence, anxiety, narrative identification, feedback, and perceived competence. It is methodological, when researchers need tools to document learning across less standardised settings. It is professional, when teachers must learn to design, orchestrate, observe, and interpret mathematical activity beyond ordinary classroom routines.

The issue therefore does not propose outdoor education, summer activities, digital tools, storytelling, AI feedback, or family involvement as simple solutions. Rather, it treats them as contexts in which the problem of continuity becomes visible and can be studied. Each contribution shows, in its own way, that mathematical meanings do not travel automatically. They need mediation. They need tasks, artefacts, representations, discussions, routines, feedback, documentation, and institutional support.

This is the central rationale of the special issue: to examine how mathematical activity can be sustained across interruptions and differences without being reduced either to school repetition or to informal spontaneity. The question is not how to export classroom mathematics unchanged into other spaces. It is how to design trajectories in which mathematical experiences can be transformed while remaining recognisably mathematical.

4. Implications for research, teaching, and teacher education

For research in mathematics education, the special issue suggests the need to study trajectories rather than isolated events. Outdoor lessons, museum visits, summer apps, family games, AI feedback, narrative pathways, and teacher education workshops acquire educational meaning when they are connected to what prepares them, what happens within them, and what follows from them. Methodologically, this requires heterogeneous data: observations, artefacts, digital traces, interviews, drawings, written productions, discussions, focus groups, and forms of longitudinal documentation.

For teaching, the issue highlights the importance of designing mathematical activities that are both situated and mathematically explicit. The outdoor environment, the story, the web app, the family game, the museum object, or the AI feedback system should not merely decorate mathematical content. They should organise attention, support reasoning, and create opportunities for representation, discussion, modelling, argumentation, and validation.

At the same time, the contributions warn against romanticising less formal contexts. Outdoor activities need scaffolding; cooperation needs mathematical guidance; family involvement needs intelligibility; AI feedback needs critical mediation; narrative engagement needs didactical structure; intercultural inclusion needs teacher awareness and institutional support. Less formal or non-classroom settings are not automatically more inclusive, more meaningful, or more effective. They become educationally significant when they are designed and interpreted with mathematical and pedagogical care.

Teachers need to be prepared to recognise mathematics in non-standard forms, to orchestrate learning across environments, to work with families and cultural differences, to use feedback formatively, and to connect situated experiences with disciplinary knowledge. The studies on Outfield Education, intercultural teacher training, and AI-mediated formative feedback show that professional development must address not only new tools and new settings, but new forms of professional judgement.

Teachers are not simply implementers of activities. They are designers and mediators of continuity. They decide what can be prepared in the classroom, what can be explored outdoors, how an experience can be documented, how students' productions can be discussed, how family participation can be made meaningful, how digital tools can support rather than replace interaction, and how culturally diverse mathematical knowledge can be recognised rather than dismissed.

At the institutional level, the issue points toward the need for stronger alliances among schools, universities, families, museums, local communities, and digital learning environments. Summer learning cannot be addressed only through homework. Outdoor mathematics cannot depend only on individual teacher enthusiasm. Family participation cannot be assumed without support. Teacher education cannot remain confined to general principles without concrete design work. Continuity requires infrastructures: shared planning time, research-practice collaboration, accessible resources, documentation tools, and institutional recognition of mathematical learning beyond the classroom.

4.1. Concluding remarks: towards resilient and situated mathematics learning

The special issue closes without offering a single model of mathematics learning beyond the classroom. Instead, it proposes a research agenda. Future work should investigate how summer interventions can sustain regular and meaningful engagement rather than last-minute completion; how schools, families, universities, museums, and local communities can collaborate without transferring responsibility unevenly to families; how outdoor and museum-based learning can be documented and evaluated without reducing their richness to standard classroom measures; how teachers can be prepared to design mathematical activities that cross spatial, cultural, and temporal boundaries; and how curricula can become more permeable to informal, semi-formal and digital forms of mathematical experience.

The challenge is not simply to compensate for interruption. It is to imagine mathematics education as a more resilient and situated field of experience. The summer break, the home, the schoolyard, the museum and the digital forum should not be treated as peripheral spaces in which school mathematics is merely applied or repeated. In the studies collected are settings in which mathematical activity is reorganised through different mediations: adult support, children's expertise, artefacts, bodily movement, feedback, narrative identification, cultural recognition, and teacher orchestration.

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