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From classroom to outdoors and back: orchestrating the field of experience of sun and shadows in mathematics education

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Abstract: This paper presents a perspective on outdoor mathematics education developed through collaborative inquiry of researchers and Italian in-service teachers from the first cycle of education. Drawing on Priest's notion of Outdoor Education and the construct of fields of experience, we conceptualize our perspective of outdoor mathematics education as a process in which students explore mathematically rich phenomena across classroom and outdoor settings through digital and non digital artefacts. The analysis draws on two data sources: a video-recorded teaching episode in a fourth-grade classroom and schoolyard, and a focus group with teachers after the implementation. The findings highlight the role of relationships, the contribution of digital and material tools, and the challenge of curricular integration. On this basis, the paper proposes the provisional notion of Outfield Education, in which mathematical learning develops in, about, and through a field of experience, connecting school learning, outdoor exploration, and students' lived worlds, within which their actual experiences unfold.

2020 Mathematics Subject Classification: Primary 35B65; Secondary 35J70, 35R09.
Keywords: outdoor education; fields of experience; instrumental orchestration; teacher professional development.

1. Days of a future past

In 2019, Bakker, Cai, and Zenger asked mathematics education researchers, through a survey, the following question: "What themes should research in mathematics education focus on in the coming decade?" [1, p. 2]. The survey, which involved a significant number of researchers from diverse countries, identified many topics. Among these, one theme in particular concerned the proposal to teach and learn mathematics in settings other than the classroom, situated in contexts that stimulate the exploration and understanding of mathematics within them:

> "Though methodologically and theoretically challenging, it is of great importance to study learning and teaching mathematics across contexts. After all, students do

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> not just learn at school; [...] Mathematics learning could also be situated on streets or in museums, homes, and other informal settings.” [1, p. 16]

As the authors point out, teaching mathematics in real-world settings outside the classroom presents methodological challenges. Wolf et al. [2] highlight that teachers often feel ill-prepared to teach outdoors, which is one reason why they make limited use of this approach in their lessons. This difficulty is confirmed by several literature reviews [3, 2], which show that Outdoor Education as a teaching and learning methodology is rarely addressed in teacher education programs, both pre-service and in-service. Exceptions can be found in contexts where Outdoor Education has long been recognized as a legitimate teaching practice, such as in Swedish universities, where pre-service teachers study outdoor activities as part of their teacher education programs [4]. The Italian context, in which this research is situated, is no exception. Outdoor Education receives very little attention in teacher education programs and, in practice, remains limited to a small number of schools that choose to specialize in these methodologies, even though, over recent years, awareness of this issue has been growing, encountering both resistance and interest, as well as a willingness to change [5, p. 21].

Bakker et al. [1] also mention a theoretical challenge, which is further emphasized by Wolf et al. [2] in their discussion of Outdoor Education. The understanding and practice of Outdoor Education are highly diverse and influenced by the sociocultural contexts in which they are enacted [6]. As a result, defining Outdoor Education and positioning it within a theoretical framework has also proved difficult [7, 8]. In the Italian context, the National Guidelines for the Curriculum of the first cycle of education (from early childhood to grade 8) suggest that a tradition of outdoor teaching is scarcely present, as it is mentioned only as a desirable practice in early childhood education or for science activities in lower secondary school (grades 6 to 8) that may specifically require fieldwork [9]. In the new proposal for the Italian National Guidelines of the first cycle of education, Outdoor Education is cited for geography and physical education as a methodology that should be promoted [10]. In conducting our research, we actually found researchers in education that, over the years, may have contributed to defining what Outdoor Education is in the Italian context [5]. On this matter of definition, we turned our attention to one encountered in various contributions to Outdoor Education research and that appears relatively free from specific sociocultural influences. We decided to proceed in this direction since we wanted to adopt a definition that could serve as a starting point for developing our own perspective on Outdoor Education for mathematics. Such definition is provided by Priest (Figure 1):

> “outdoor education is an experiential process of learning by doing, which takes place primarily through the exposure to the out-of-doors. In outdoor education the emphasis for the subject of learning is placed on RELATIONSHIPS, relationships concerning people and natural resources.” [11, p. 13]

In this definition, we find several connections to our own field of research, namely mathematics education. A first theme that is particularly significant for us is the idea that learning takes place primarily outdoors, but not exclusively. This point is important because it opens the discussion on one of the critical issues of outdoor or informal experiences, which often risk remaining isolated, wonderful activities from which only a fond memory endures over time [12], rather than becoming part of a broader educational trajectory. It also raises the issue of how to create bridges between in-school and out-of-school learning [13]. In this regard, Rickinson et al. [3] argue for the need to rethink school curricula by providing more space for outdoor experiences, as well as guidelines and resources for teachers to integrate meaningful outdoor activities into classroom curricula.

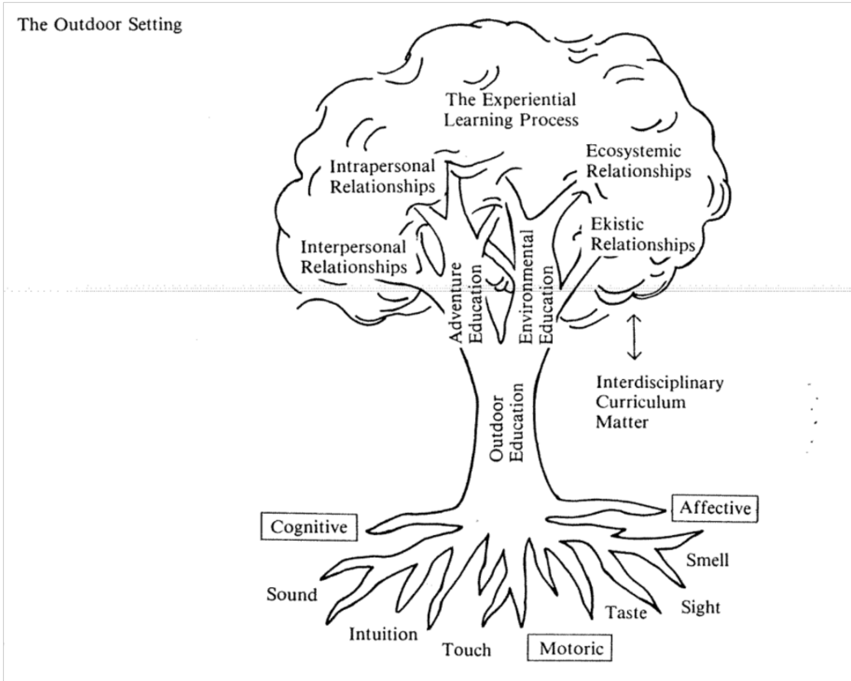

Figure 1. A drawing representing a tree as a metaphor to describe Outdoor Education. In the soil the cognitive domain and the senses through the learning happens, the outdoor setting as the Sun interacting with the leaves of the tree in which is situated the experiential learning process and with it the relationships between people and the environment, in the air the interdisciplinarity matter [11, p. 15].

The definition also emphasizes relationships: between individuals who learn together and between individuals and the environment in which learning takes place. Learning occurs through meaningful experiences, with an approach that resonates with Dewey's notion of "learning by doing" and, more broadly, with the tradition of pedagogical activism. This focus on relationships and on the surrounding environment is also emphasized by some of the researchers who responded to the survey by Bakker et al. [1], highlighting the societal goals of mathematics education:

> "Apart from instrumental goals of mathematics education, some emphasized goals related to developing as a human being, for instance learning to see the mathematics in the world and develop a relation with the world." [1, p. 7]

Learning to see mathematics in the surrounding reality thus appears to be a key theme. However, from the perspective developed in this section, the main question that emerges and that we wish to address is the following: What is our idea, as an Italian community of inquiry composed of mathematics education researchers and mathematics teachers, of outdoor mathematics education activities? To better understand our vision, we sought to design and implement mathematics programs that included both outdoor and classroom activities, with the aim of creating continuity between traditional curricular work and outdoor experiences. In reflecting on the feasibility of such integration, we as researchers also created opportunities for discussion and reflection with teachers after their implementations, in order to gain a deeper understanding of the work carried out and to frame it also theoretically, so that these activities could be meaningfully shared within both the worlds of mathematics education research and teaching.

This paper should therefore be read as a pilot and exploratory contribution, somewhat unusual in genre. Rather than presenting a primarily empirical study aimed at systematic generalization, it offers a theoretically oriented reflection emerging from an empirical experience conducted with teachers. The data are used to illustrate and refine a conceptual proposal for outdoor mathematics education. The paper first situates this proposal within the literature on Outdoor Education, Fields of Experience, and the Instrumental Approach; it then presents the collaborative inquiry with teachers, analyzes one implementation episode and a post-implementation focus group, and finally introduces the provisional notion of Outfield Education, discussing its limitations and possible future developments.

2. You are here: our perspective on outdoor mathematics education

Just as maps of public places help people orient themselves by clearly marking "you are here" with an ideolocator, we too seek to position our work and theoretical perspective within the existing literature on outdoor mathematics education. Our approach to outdoor mathematics education stems from bringing together Priest's [11] definition of Outdoor Education (OE) — understood as an experiential process of learning by doing, in which emphasis is placed on relationships (see Figure 1) — with the notion of fields of experience (FoE), understood as "an area of human culture identifiable by learners as part of their (actual or potential) experience, with specific characteristics that make it (under the guidance of the teacher) suitable for activities in mathematical modeling, mathematical problem solving, and the construction and development of mathematical concepts" [14]. In this respect, Priest's definition differs from earlier ones, such as that of Donaldson and Donaldson [15], who defined OE as education "in, about, and for the outdoors". With Priest, OE begins to be framed more explicitly as a resource for teaching and learning in school: the focus is no longer only on the outdoor context itself as the object of study, but also on the educational possibilities it opens up, including interdisciplinary ones. From this perspective, the outdoor setting can offer meaningful insights for learning across domains, including mathematics. Similarly, as we will discuss later, the FoE can become the context within which learning unfolds, both outdoors and in continuity with classroom activity, offering a space for the exploration and development of mathematical ideas.

This choice of mixing OE and FoE is grounded in the fact that the concept of FoE serves as a reference point in the National Curriculum Guidelines [9], where it is defined in terms that are less specifically oriented toward mathematics and is presented as an approach to teaching in early childhood education. Moreover, when discussing teaching practices based on FoEs, outdoor education is explicitly suggested [10]. Within the Italian tradition, studies on FoEs already exist, such as those focused on sun shadows [14], which can retrospectively be interpreted as proposals for OE, since they included some outdoor activities aimed at the exploration of a FoE. Seeking a FoE that is (or could be) consistently part of the learner's lived experience creates strong connections with Priest's view of OE, which is defined first and foremost as a process of experiential learning and, in this case, unfolds through the experiential study of the FoE.

We believe that artifacts can play a crucial role in the experiential process of exploring and understanding a FoE and the mathematics embedded within it. To define these artifacts and the ways in which they are used, we draw on the Instrumental Approach [16], which distinguishes between an artifact, that is, the given object, and an instrument, a psychological construct that exists "when the subject has been able to appropriate it for himself, has integrated it with his activity". The instrument thus becomes an extension of the body, and its construction is described as a process of instrumental genesis. According to Rabardel [16], for an artifact to become an instrument, it is necessary to develop utilization schemes, that are schemes that organize activities with the artifact in order to solve specific tasks. He distinguishes between usage schemes, oriented toward the management of artifacts, and instrumented action schemes, oriented toward carrying out specific tasks. Going deeper in the theory, instrumental genesis can be also described as the combination of two processes: instrumentalization, directed toward the artifact, and instrumentation, directed toward the subject.

A theory rooted in the instrumental approach that explains how teachers can guide their students processes of instrumental genesis has been defined as instrumental orchestration [17]. This refers to the teacher's intentional and systematic organization and use of available artefacts within a given learning environment and mathematical task situation, with the aim of guiding students' instrumental genesis [17, 16]. Three key components define instrumental orchestration (IO): (1) the didactic configuration, referring to the arrangement of artefacts within the learning environment; (2) the exploitation mode, which concerns how the teacher chooses to make use of a given configuration in order to serve specific didactic intentions; and (3) the didactical performance, encompassing the spontaneous, in-the-moment decisions made by the teacher during instruction [18]. The ways in which the teacher guides this process of instrumental genesis can be further understood through the notion of semiotic mediation [19], which describes how the teacher, by building on students' artefact-mediated activity, directs attention toward mathematical signs and meanings in a collective setting. The guidance of students' instrumental genesis is closely related to OE, since artifacts always carry a social dimension: "Tools are not passive, they are active elements of the culture into which they are inserted" [20, p. 58]. In this direction, it has been argued that schemes always have both an individual and a social aspect, since they can be social schemes, "elaborated and shared in communities of practice and can give rise to appropriation by the subjects, even come under training processes" [21]. From this perspective, one of the main actors are the relationships that develop among a group of individuals and within a specific environment while exploring a FoE and its mathematics through the use of tools.

Our conceptualisation of outdoor mathematics education, which emerges from the networking of these theoretical perspectives through combining [22], seeks to describe and understand a scenario in which mathematics teachers attempt to orchestrate artefacts, tasks, social interactions, and learning environments in support of the exploration and comprehension of a specific FoE. Particular attention is given to the teacher's action, which also involves orchestrating different settings — both in-school and out-of-school — in order to pursue specific educational goals while supporting the experiential learning process. In this sense, the challenge lies in recognizing what is best developed within the classroom and what, instead, is more effectively learned through direct experience in the outside world [23].

2.1. The outdoor mathematics education horizon

In mathematics education, out-of-school learning is a relatively new field compared with science education, although awareness of its crucial role in a range of contexts — from museums and outdoor settings to everyday life — is steadily increasing [24]. In science education, Eshach [13] is one of the scholars who addresses an issue that we also in the introduction considered central: establishing connections between in-school and out-of-school learning. His approach distinguishes between non-formal learning, which is structured and takes place in institutions outside school, and informal learning, which refers to situations that arise spontaneously in everyday life, such as within the family, the neighborhood, and similar contexts [13]. This distinction also aims to bring greater clarity to the use of the terms out-of-school, informal, and non-formal, which are often used interchangeably, thereby generating considerable confusion.

In mathematics education as well, the concept of informal mathematics education has been defined as a voluntary educational experience characterized by fluid boundaries between disciplines and by the absence of traditional forms of assessment [25]. Nemirovsky et al. [25] introduced the term informal mathematics education in order to distinguish it from the everyday, spontaneous ways in which people encounter mathematics in daily life. For example, they considered museums to be intentionally designed informal mathematics learning settings because of their structured schedules, the presence of educators, and the availability of technologies and tools to support mathematical learning. Given these characteristics, their perspective appears closer to Eshach's concept of non-formal learning than to his definition of informal learning.

The work that we describe in the next section seeks to address how to connect out-of-school mathematics learning experiences with classroom curricular experiences. Since the proposed activities are carried out by students during school hours, in mathematics lessons, together with their teacher, and are integrated as much as possible into the classroom curriculum, they are, on paper, quite distant from the notion of informal learning described by Nemirovsky et al. [25], particularly with regard to the voluntary nature of students' participation. Students are assigned a task to solve, which guides them toward specific learning goals; the pathway is therefore clearly planned both in the classroom and outdoors. Consequently, according to Eshach's [13] categories, our idea of outdoor mathematics education, as we will also see in a teaching episode discussed in the analysis, is more appropriately classified as a form of formal education, but one that also makes use of a setting out-of-school in order to foster a less formal reflection on the mathematics of the explored FoE and to prompt a more genuinely experiential learning process [11]. One of the aims of this approach is also to create stronger connections between experiences conducted in informal settings and classroom lessons, thus resembling explorations of the ways in which informal and formal learning experiences can be connected, a topic that is already prominent in mathematics education research [26, 27].

Working with FoEs can also open up a dialogue among different scientific disciplines, an aspect that has likewise been identified as a topic of interest in mathematics education research:

> "Many responses can be characterized as highlighting boundary crossing [28] with disciplines or communities outside mathematics education, such as in science, technology, engineering, art, and mathematics education (STEM or STEAM); parents or families; the workplace; and leisure (e.g., drama, music, sports)." [1, p. 7-8]

The interdisciplinary nature of STEM education, in which different disciplines are intertwined within a broader academic strand, also presents mathematics teachers with a new challenge: mathematics is no longer approached as an isolated subject, but in dialogue with other disciplines. Among these challenges, digital technologies are especially pressing, as they create new learning environments with countless possibilities, but also with risks that must be discussed and addressed. From our perspective, since today's mathematics teacher is required to face these new challenges in both primary and secondary education [1, 10], a convincing response lies in seeking a balance between the educational goals associated with natural and digital learning environments (Figure 1). According to some studies, the combination of technology use and outdoor teaching practices has a positive impact on both dimensions. Students who are accustomed to spending substantial time outdoors do not seem to reach alarming levels of media use, as they tend to choose certain tools only when they are truly necessary [29]. Moreover, OE engages directed attention through successive action planning, which helps inhibit the impulse toward distraction that technology can intensify; for this reason, it has beneficial effects on cognitive and emotional functions, as well as on memory and learning [30]. At the same time, the possibility of enriching an already multisensory outdoor experience through technological mediation is encouraged, provided that the experience remains stimulating in terms of whole-body engagement [31].

Within mathematics education research, Cahyono and Ludwig [32] proposed an activity that can be implemented outside the classroom through the use of digital technology. Students were asked to solve math trail tasks distributed throughout a city using a mobile application and a map; for example, they were asked to estimate the base area of a historical building, a task that required them to identify the geometric shape of the base and measure its dimensions in order to calculate its area. Similar examples can also be found in the activities proposed through MathCityMap and presented on its website, a powerful application that also offers opportunities for teacher education activities [33]. These studies represent forms of outdoor mathematics education in which digital technologies are involved, and, in this respect, they show similarities with the work we conducted. In our study, we view mathematics as guiding a broader interdisciplinary discussion capable of integrating multiple perspectives and insights from different fields, as in a STEM approach, aimed at uncovering how a specific FoE works.

Bakker et al. [1] also make clear how boundaries with other learning environments — such as the personal sphere and leisure time — can be crossed, fostering connections between mathematics education and other practices. In this regard, an approach such as Outdoor Education — which places strong emphasis on relationships — encourages the involvement of multiple stakeholders, including actors beyond the classroom, as will be shown in the examples discussed in the following sections (see Figure 2).

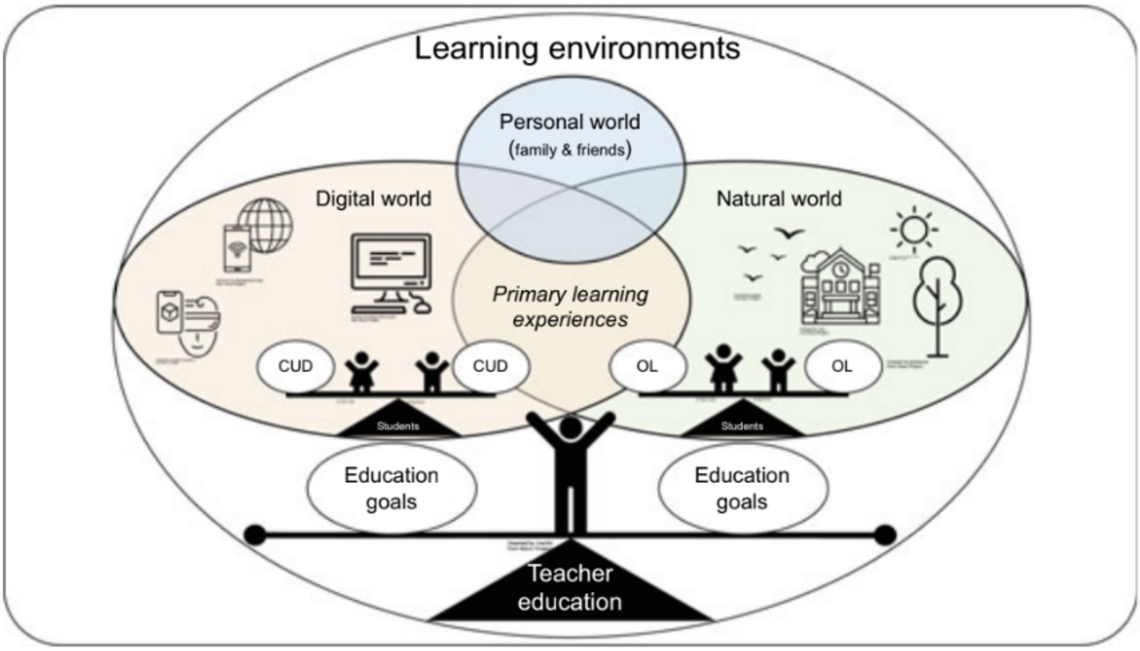

Figure 2. The teacher, supported by teacher education, has to find the right balance between the education goals for the natural and the digital learning environments. Together and in interaction with these two must be considered also the personal world, which includes family, friends and leisures [2, p. 260]

3. How our inquiry started

The first two sections of this paper outlined our perspective on outdoor mathematics education and situated it within the existing literature. Drawing on specific theoretical frameworks, we proposed an approach to mathematics education that emphasizes continuity between in-school and out-of-school settings. These theoretical considerations led us to formulate two research questions:

RQ1 How can the networking of Fields of Experience, Outdoor Education, and Instrumental Orchestration characterize and interpret outdoor mathematics education activities across classroom and outdoor settings?

RQ2 What do classroom implementation and teachers' post-implementation reflections reveal about the affordances and challenges of this approach?

To explore this theoretical proposal in practice, we worked with a group of Italian in-service teachers from the first cycle of education, spanning Grades 1 to 8. More specifically, the group consisted of 16 mathematics teachers from the same school, working across both primary education (Grades 1 to 5) and lower secondary education (Grades 6 to 8). These teachers had been collaborating for several years with researchers from the University of Eastern Piedmont, together forming a community of inquiry in the sense described by Jaworski [34]. The collaborative processes through which teachers and researchers learn from their own practice and from shared reflection constitute a fundamental aspect of such communities of inquiry. Through self-reflection and the exchange of perspectives, participants have the opportunity to develop new knowledge and to discuss both theoretical issues emerging from mathematics education research and questions arising from teaching practice, thereby fostering critical reflection on their professional work.

Methodologically, the study is framed within an Educational Design Research perspective [35], as it investigates a designed educational intervention in a real school context while also contributing to the refinement of both theoretical understanding and educational practice. In line with this approach, teachers and researchers engaged in dialogue around two closely related issues: how outdoor mathematics activities can be designed and implemented within the FoE of sun shadows, and how such work can contribute both to outlining a theoretical conception of outdoor mathematics education and to informing the development of teacher education pathways in outdoor mathematics education. Teachers designed outdoor mathematics education activities for their classes and implemented them within a framework of continuous dialogue and exchange with the researchers. From this perspective, implementation was not considered merely a context for data collection, but rather a site in which theoretical assumptions, didactical choices, artefacts, and classroom practices were tested and progressively refined.

The study therefore pursues a twofold contribution. At the theoretical level, it examines how the networking [22] of Fields of Experience, Outdoor Education, and Instrumental Orchestration can support the interpretation of mathematical activity across indoor and outdoor learning environments. At the practical-design level, the broader research project aims to develop principles for organizing teacher education pathways that support teachers in designing, enacting, and revising outdoor mathematics activities. While this second level belongs to the wider Educational Design Research trajectory, the present paper focuses specifically on the implementation and post-implementation reflections, showing how the analysis of classroom events and teachers' accounts can inform the progressive refinement not only of the educational design, but also of theoretical perspectives on education. To this end, after the implementation phase, focus groups with volunteer teachers were proposed by the researchers in order to reflect more deeply on what had concretely taken place during the classroom and outdoor implementation. Within this ongoing collaborative work with teachers, aimed at fostering richer opportunities for students' mathematical learning, the researchers can be described as didacticians[2]:

> "people from the university with knowledge of research and theory in the didactics of mathematics, interested to work with teachers to promote better opportunities for mathematics learning in classrooms." [41, p. 623]

For this inquiry, we chose to work within the FoE of sun and shadows. As highlighted by previous studies [14], this context offers rich potential for the emergence of mathematical concepts. A concept in mathematics as defined by Vergnaud [36] may be conceived and described as a dynamic system made up of three components: *reference situations*, which give meaning to a concept and can be recalled and used to reason or solve problems (e.g., noticing

[2]What we have so far referred to as researchers are, in light of our theoretical framework, now termed didacticians; however, although the two terms are sometimes used interchangeably in this paper, they refer to the same people within the community of inquiry.

that a shadow changes length during the day); *invariants*, that is, implicit ideas such as rules or relationships (e.g., the shadow depends on the position of the sun and the object); and *representations* (e.g., drawings, gestures, words). For instance, proportionality and ratio can be explored by examining the relationship between the height of an object and the length of its shadow; similar triangles can be studied through comparisons of shadow triangles; angle measurement becomes relevant when calculating the sun's elevation or tracking the movement of shadows throughout the day; and the concept of parallelism can also be meaningfully introduced. Moreover, this FoE provides a suitable context for science education activities in dialogue with mathematics, with mathematics acting as an instrument for developing a deeper understanding of these natural phenomena. In this way, it becomes possible to explore how changes in shadows throughout the day and across the seasons are related to the position of our planet and its movement around the sun. This aspect resonates with the interdisciplinary nature typically associated with OE, as well as with the idea that OE "is a matter of relationships", particularly through the exploration of ecosystemic relationships connected, in this case, to the Earth's rotation and revolution, and to the ways in which our lives and the environment in which we live change across the seasons in relation to this Sun–Earth relationship, thereby affecting our relationships with other people and with the environment.

Compared with earlier studies [14], in which teachers and researchers also worked together but with a particular focus on students' perspectives and responses to mathematics activities on sun and shadows, this research explores the same FoE with a specific focus on the perspectives of in-service teachers who had never previously engaged either with such teaching methodologies or with this particular FoE. More specifically, we seek to discuss with them both the potential and the challenges of teaching through experiential learning and, in particular, the difficulties teachers may encounter when teaching mathematics through the shadows cast by the sun, both in the classroom and outdoors. In this article, we focus on the questions highlighted above, but as said earlier this research work within a community of inquiry has also led to a further reflection that extends beyond previous studies: how to design effective teacher education pathways on these topics.

In the next section, we present two types of collected data: a teaching experiment and a focus group that involved both teachers and didacticians. The teaching experiment was led by PT1 (Primary Teacher 1), a teacher from the community of inquiry who volunteered to be filmed with her fourth-grade class, together with the researcher present to observe and support the teaching and not only to record the lessons. The teaching experiment comprised approximately 18 hours of lessons in total. From these video recordings, we selected a number of excerpts for this article that are particularly useful for highlighting key aspects of the three theoretical frameworks — Instrumental Orchestration, Fields of Experience, and Outdoor Education — as they emerge in synergy and in ways that complement one another during the teaching process. We provide a narrative description and a few excerpts accompanied by photographs from one selected lesson of this teaching experiment, intended to illustrate key aspects of our theoretical proposal in a specific moment rather than to exhaust the empirical material on which the broader research draws.

In analysing this episode, we focus on aspects that are particularly significant in relation to our vision of outdoor mathematics education. More specifically, we highlight the three characteristic elements of instrumental orchestration — *didactic configuration*, *exploitation mode*, and *didactic performance* [18]. Since instrumental orchestration concerns the ways in which a set of instruments is organized and exploited within a learning environment, we examine how these elements change across different learning environments and how relationships between students and the environment are fostered [11] through specific orchestration choices aimed at supporting understanding of the phenomenon of sun shadows through particular activities. Consequently, we focus on the ways in which the FoE of sun shadows, together with some of the mathematical concepts embedded within it and their components [36], are explored through the instruments orchestrated by the teacher, as well as on the relationships among all the agents involved. This episode is also intended to provide an overview of the types of activities proposed and of how they were carried out both inside and outside the classroom.

The analysis of this episode is conducted through qualitative content analysis [37], which is used to identify the three characteristic elements of orchestration [18], the components of the mathematical concepts explored within the FoE [36], and the relationships among the agents involved in the activities [11]. The aim is not merely to categorize these elements with a deductive coding approach, but to show how they interact with one another. In particular, we seek to show how the teacher's orchestration is guided by an understanding of the mathematics embedded in the FoE, how it engages different components of the concepts being explored, and how it fosters relationships among all the people involved in the activity and with the environment in which it takes place, thereby supporting a collective learning-by-doing approach — constructed together with the whole class in interaction with the environment — that is characteristic of Outdoor Education [11]. Going deeper in the analysis searching for the interactions between these three theories we highlighted two categories that will be discussed in "Analysis I": *the relational mathematization of the phenomenon* and *the role of artefacts across environments.*

In the "Analysis II" section we analyze excerpts (transcribed by the author) from reflections shared by teachers during a video-recorded focus group (one of the two focus groups implemented), involving two didacticians and six teachers (4 of primary school, 2 of low-secondary school) of the community of inquiry that proposed themselves as volunteers. The focus group has been proposed and conducted by the didacticians at the end of the implementation phase. The focus group was organized as a structured collective reflection, guided by three broad questions concerning the unfolding of the implementation, teachers' and students' experiences, the use of tools inside and outside the classroom, and possible modifications of the proposal. The discussion of the group was guided by three questions:

1. How did the implementation unfold in the classes? How did you experience it as teachers, and how did your students respond to this Outdoor Mathematics Education initiative?
2. We used various tools in the activities. Some were suggested by us, such as the app, while others were selected and used by you. We would like you to tell us about the reasons that guided your choices regarding the types of tools adopted and the ways in which you used them inside and outside the classroom. Would you make the same choices again, or do you think some aspects should be changed?
3. What would you keep unchanged, and what would you modify in the proposal?

The didactician who organized the focus group (the author of this paper) had also prepared a checklist for himself including all the topics he intended to address during the discussion. If any of these topics did not emerge spontaneously, he intervened beyond the three main questions by providing more direct prompts.

The analysis of the excerpts of the focus group was conducted through an abductive thematic qualitative content analysis [38, 37, 39]. The transcript was first divided into meaning units, corresponding to teacher turns or short sequences in which participants reflected on a specific aspect of the experience. These units were initially coded descriptively, remaining close to the content of teachers' accounts. In a second step, related codes were grouped into broader thematic areas (see Table 1). This process led to the construction of three main areas: (1) relationships among participants and with the environment; (2) the digital world and the use of instruments; and (3) difficulties related to teaching practice and curriculum integration. The analysis was abductive because these themes were not treated as purely inductive findings, nor as deductive applications of pre-existing theory. Rather, they were constructed through a movement between teachers' reflections, the two categories identified in "Analysis I" — *relational mathematization of the phenomenon and role of artefacts across environments* — and the networking through combining [22] of Outdoor Education, Fields of Experience, and Instrumental Orchestration. In this sense, the focus group was not used to validate the classroom analysis, but to examine how teachers expanded, problematized, and recontextualized the issues that had emerged during implementation.

Focus-group thematic area	**First-cycle codes**	**Link with Analysis I**	**Theoretical interpretation**
Relationships among participants and with the environment	Student collaboration; mutual support; autonomy; increased observation; familiar places; family involvement; school community; local territory	Expands *relational mathematization of the phenomenon*	OE: relationships and environment. FoE: continuity between lived experience and mathematical phenomenon. IO: teacher organization to foster social and spatial relations
Digital world and use of instruments	App engagement; construction of gnomons; physical tools; digital tools; students' different strengths; participation through tools	Expands *artefact-mediated continuity across environments*	IO: orchestration of tools. FoE: artefacts mediate access to the sun-shadow phenomenon. OE: tools connect school activity with students' lived worlds
Difficulties and curriculum matter	Teacher uncertainty; need for colleagues; need for didacticians; weather constraints; time pressure; redesign; longer duration; vertical curriculum	Emerges from teachers' reflection as a third dimension, only partly visible in Analysis I	IO: flexibility and didactical performance. FoE: progressive development of concepts across grades. OE: feasibility and sustainability of outdoor practice

Table 1. Description of the process of abductive thematic qualitative content analysis

4. Analysis I: the teaching episode

The first episode we wish to present began in the classroom, where PT1 implemented the activities previously outlined during the collaborative design process carried out within the community. In this opening session, she introduced the gnomon[3] and encouraged students to share their prior knowledge and conjectures about its function. To do so, she organized a didactic configuration that included a photograph of the gnomon and its shadows projected on the interactive whiteboard, a tangible model consisting of a straw positioned perpendicularly on a paper plate serving as its base, which students were later asked to replicate, and two drafting triangles, later used to check that the straw was indeed perpendicular to the base.

[3]namely, a nail or stylus whose shadow is used to indicate the hours on a sundial

PT1: (after two groups have aligned the shadows of their gnomons to form a what they called "highway of shadows") Shall we try to join them all together? (involves the third group as well. The children join the shadows of all the gnomons together)

PT1: I see that you put these three gnomons side by side. What can you observe? How are those three shadows?

S1&S2: teacher we understood that it forms a drafting triangle. (S2 shows to the teacher first with his hand then with a real drafting triangle) [. . .]
PT1: So how is the shadow formed?
S2: The sun's rays are blocked by the gnomon and create the shadow

Figure 3. The Figure is divided in three rows. In each row you see a photo of the activity and selected excerpts of teachers and students. First row (A) the students putting gnomons side by side; in the middle row (B) the "highway of shadows"; on the last row (C) a student using a drafting triangle to represent the shadow triangle

Although some moments may be interpreted as instances of didactic performance, insofar as she shifted orchestration types in ways not anticipated during the design phase, this became particularly evident when students first tested the instruments in the classroom by opening the curtains to let sunlight in and then asked whether they could go outside to test their gnomons. At that point, the teacher took up this stimulus and modified the lesson plan as it had been defined during the design phase, anticipating the outdoor component already in the first lesson.

As a result of this change, the teacher had to redefine the didactic configuration on the spot. The students and the teacher moved outdoors to the school courtyard — a new learning environment — bringing with them the personal gnomon each student had constructed and the two drafting triangles provided by the teacher. PT1 divided the class into three working groups and assigned the following task: "Working with your group, position the gnomon and observe the shadows to understand what happens". The groups were then free to explore the phenomenon outdoors and to use the tools in ways they considered most appropriate for addressing the task. The teacher's initial exploitation mode consisted of circulating among the three groups to offer prompts and suggestions, such as using her own shadow to cover that of the gnomon or redirecting students' attention to their own shadows and to those cast by objects in the courtyard, such as the gate, the tree, and a wall. In this way, the class was guided in exploring the FoE by reflecting on the relationships among the elements present in the environment at that moment [11], particularly on the shadows they cast and on the relationship between those shadows and their source — the Sun. The following excerpts illustrate how the teacher supported students' explorations, thought in the moment to be of interest for all the students, through a series of guiding questions.

We can observe how, starting from students' spontaneous ways of exploring sun shadows through these relationships, the teacher drew on representations that she considered significant for understanding the phenomenon. For example, when some students coordinated their gnomons to produce a single continuous shadow (Figure 3A), thereby enacting a spontaneous utilization scheme, this became a pivotal moment of didactic performance: prompted by students' actions, she shifted from monitoring small-group exploration to orchestrating a whole-class discussion. She may have recognized an ongoing process of instrumentation as students coordinated their instruments, which in turn created a further opportunity to direct their attention to the direction of the shadows.

Subsequently, when she observed that some students were placing their gnomons side by side to compare the shadows (Figure 3B), she took up this utilization scheme and reoriented the whole class's attention. In doing so, she not only extended the reflection on shadow length by inviting comparisons across different gnomons, but also appeared to shift the focus from the direction of each individual shadow to the recognition of the parallelism of shadows at the same moment of the day, thus continuing the observation of invariant elements. At first, this idea did not seem readily available to the students. However, by building on their spontaneous actions, the teacher was able to make this mathematical feature more visible within the collective discussion. In this action, the teacher worked as a semiotic mediator [19]: she built on what some students had independently begun to represent, seeking to involve the entire class in reconstructing that representation together and thereby extending an understanding of the parallelism of shadows — which initially seemed to be grasped only by a few students — to the rest of the class. In this way, the teacher created a network of connections among the ideas generated by students during their exploration, putting these interpersonal ideas [11] into relation with one another in order to synthesize, through dialogue around shared observations, an understanding consistent with what the phenomenon reveals from a mathematical perspective.

A similar situation occurred when two students began constructing the shadow triangle, also using the drafting triangle (Figure 3C) as a representation capable not only of modelling the phenomenon, but also of offering insight into mathematical concepts such as angles, triangles, and similarity. This process can be interpreted both as instrumentalization — in the ways the students acted on the drafting triangle, adapting its use beyond what is typically expected for this artifact — and as instrumentation, insofar as this use shaped their activity by supporting the visualization of a model of the shadow triangle.

The episode can therefore be interpreted through two main cross-theoretical categories. The first is the relational mathematization of the phenomenon: students' outdoor exploration of sun shadows became mathematical as they established relationships among bodies, gnomons, environmental objects, shadows, and the sun [11]. These relationships were made available by the outdoor environment, belonged to the FoE of sun shadows, and were progressively oriented by the teacher's orchestration, which acted as a form of semiotic mediation by selecting the representations most suited to the educational goals she had in mind and directing attention toward mathematical meanings such as direction, shadow length, parallelism, and the modelling of the phenomenon through the shadow triangle.

The second category concerns the role of artefacts across environments: the gnomon, the drafting triangle, classroom representations, students' bodies, and the outdoor shadows formed an artefact ecology that connected the classroom introduction with the outdoor investigation, later culminating, within the teaching experiment, in a process of formalization back in the classroom. Through this ecology, artefacts changed function, supported students' spontaneous utilization schemes, and mediated the transition from perceptual exploration to mathematical representation.

Taken together, these two categories show that the implementation was not simply a movement from inside to outside, nor a straightforward enactment of a planned task. Rather, the teacher's didactic performance consisted in recognizing students' exploratory actions and using them to connect outdoor experience, artefact use, and the construction of mathematical meaning. For this reason, these categories provided the basis for the author of this paper in structuring the subsequent focus group, in which teachers and researchers were invited to reflect on relationships, tools, and the actual integration of the proposal into the curriculum.

5. Analysis II: inquiry over the implementation

Following the implementation, teachers were invited to participate voluntarily in a structured reflection session organized as focus groups. The two categories that emerged from the analysis of the implementation — namely, *the relational mathematization of the phenomenon* and *the role of artefacts across environments* — were subsequently used to structure the post-implementation focus group, allowing us to examine whether teachers' broader reflections on the experience also recalled these categories. Their reflections were ultimately organized into three macro-areas: (1) relationships among participants and with the environment; (2) the digital world and the use of instruments; and (3) difficulties related to teaching practice and curricular integration. The first two macro-areas echo the categories that emerged from the implementation analysis, while the third introduces a further issue of interest, namely the teachers' perspective on the feasibility of integrating such practices into their teaching and into the class curriculum. These macro-areas do not replace the theoretical categories developed from the implementation; rather, they provide a second interpretive layer through which the networking of Outdoor Education, Fields of Experience, and Instrumental Orchestration can be further specified.

With regard to the participants involved in the focus group, the term PT stands for Primary Teacher and ST for Secondary Teacher. PT1 is the same teacher featured in the teaching episode described in the previous section. The excerpts are drawn from the broader discussion and are organized according to the three main categories.

5.1. Relationships among participants and with the environment

> PT2: They worked very well together. [...] Just as PT1 mentioned earlier, while one person might be taking measurements, another would step away — but only to explore our yard and observe. [...] I've noticed that since we started this project, they've become more observant.

Based on the reflections shared by the teachers in the focus group, it appears that, in their interpretation of the classroom experience, the proposed activities helped strengthen relationships among the children [11, 3] by fostering spontaneous collaboration and mutual support during exploration. Again from the teachers' perspective, this process seems to have been intertwined with the development of greater student autonomy, not only within the context of the educational activities themselves, but also in broader patterns of behaviour. In particular, the teachers reported observing in the children a stronger inclination to attend to their surroundings and a more active and curious attitude extending beyond structured work sessions.

> PT3: As for the environment and the children, it's a setting they're already familiar with. Sometimes, in the afternoons, they climb up to the park to observe and engage in hands-on activities there.

The role of the environment emerges as a crucial dimension of the experience, extending beyond the schoolyard alone and becoming something more than just a space in which to organize artifacts [18]. In the teachers' view, the value of the explored contexts lies not only in their availability as outdoor spaces, but also in their capacity to foster a meaningful relationship with places that are aligned with the FoE under investigation and, at the same time, already embedded in the children's everyday lives. From this perspective, a particularly significant role was attributed to the park, described by the teachers as a themed park located near the school and equipped with instruments for measuring time through shadows. The choice of this location appears especially meaningful because it constitutes a space closely connected to the topic being explored while also remaining familiar to the students, who visit it outside school hours as well. According to the teachers, the exploration of the park, further enriched by the explanations provided by the park experts about the various instruments present, strengthened the link between educational experience and lived experience [14], reinforcing continuity between school learning and students' everyday relationship with their local area. Similarly, the selection of other spaces, such as the town square, is described by the teachers as a choice that, although at times motivated by practical constraints — such as limited sun exposure or the small size of the schoolyard — maintains a strong connection with places that form part of the children's actual daily experience [14]. In this sense, their reflections suggest that the environment is not treated merely as a logistical backdrop for the activity, but as a constitutive component of the FoE, capable of supporting forms of exploration that are richer in meaning precisely because they are rooted in spaces that already belong to the students' lives.

> ST1: I can say that I was particularly impressed by some students who aren't the usual high-achievers who typically participate in class; I actually saw them actively participating, taking charge, and making a meaningful contribution to the activity that had been proposed. [...] At first, I was worried that if I took the students outside, not everyone might pay attention. [...] Most of them understood that this was an educational activity, so their behavior outdoors was just like it was in the classroom.

> PT1: I felt the same way whenever I saw some students wandering off. . . I'd already be giving them the death stare, but maybe they were just exploring another part of the courtyard.

These comments also appear to echo some of the concerns that often accompany teachers' initial perceptions of activities conducted outside the classroom [3]. Their words reveal a tension between, on the one hand, the initial impression that children may be less engaged or less visibly "at work" than in more traditional school settings and, on the other hand, the subsequent recognition that meaningful learning processes are nevertheless taking place throughout the experience [11]. From the teachers' perspective, what may initially appear to be reduced concentration or more dispersed participation seems instead to point to different forms of engagement — freer and less controlled, yet no less relevant in terms of learning. In this sense, their reflections, grounded in students' actual behaviour during the proposed activities, challenge the assumption that learning must necessarily manifest itself through forms that are immediately recognizable as academic. Rather, they suggest that, in outdoor settings, learning can also take shape through exploratory and participatory dynamics that are less rigidly structured.

> PT1: There were parents from the preschool coming out with their children, and they often stopped to ask what we were doing.

> PT3: It happened to me, too — a mom asked me why we were doing this activity, and I didn't think the child had mentioned it at home because he's a bit reserved. So when she told me, I was really happy to hear that he'd opened up to.

> PT4: As for the families, the children told their parents all about what they were doing. In our school complex, we usually invite families to visit two or three times a year to show them the projects we're working on. They came to see these activities

> once we had finished them, and the principal — who is always there — joined them, and the families were truly shocked. They hadn't expected anything like that.

The experience also appears to have extended beyond the school setting, involving students' family lives as well (Figure 2)[2]. According to the teachers, the children tended to speak at home about what they had experienced during the activities and, in some cases, spontaneously recreated similar experiences outside school, a tendency encouraged by the fact that the FoE of sun shadows lends itself to exploration across many contexts of daily life. From the teachers' perspective, this continuity between school and out-of-school life generated growing interest among parents, who sought more information both about how the proposed activities were carried out and about the educational rationale underpinning them. In this sense, the teachers view the project as an opportunity to bridge students' personal and school lives, fostering stronger connections among lived experience, learning, and family involvement.

> ST1: And then we asked for the help of other teachers of the class who basically assisted during the shadow measurements, at times when we math teachers couldn't be present[...] I can say that we involved the entire school, every single person...

> ST2: Even the school staff who told us, "Look, teacher, it's starting to rain, you should remove the gnomon..."

> PT2: The staff member helped us a lot with the activities, and everyone was talking about the gnomon. A colleague went on vacation to the mountains and sent me a photo of a meridian.

> PT4: And of course, my colleagues helped me a lot because even when I wasn't in class, they took the kids outside to take measurements, and the other classes got interested too. Once, the third and fourth graders came out with us to see what their classmates were doing and asked a few questions.

As these accounts show, support among colleagues — including those working at the same grade level but in different subject areas — appears to be a key factor in the success of certain activities. The teachers also emphasize how the project strengthened relationships among multiple school stakeholders by involving school staff and teachers from other classes. Finally, according to their reports, the outdoor activities also sparked curiosity among students who were not directly involved and who, on some occasions, asked to take part.

5.2. The digital world and the use of instruments

> ST1: I'd like to echo what ST2 said earlier: for secondary school students, it's definitely been a positive experience. It really brought out some of the students' abilities that don't always come to the surface in math class. One girl who doesn't usually speak up surprised me with her analysis of the app.

> ST2: I agree because, in fact, for middle school students, digital technology serves as the kind of stimulus that they might have experienced through hands-on activities in elementary school.

The use of the digital app appears to have been a factor that fostered student engagement, particularly among secondary school students. According to the teachers, the use of a digital tool helped draw students into the proposed activities by activating a dimension of their actual experience that is especially meaningful to them [14]. From their perspective, digital technology is not only an environment that has become pervasive in contemporary life, but also a significant component of students' lived worlds (Figure 2), capable of shaping how they participate in, observe, and relate to the proposed experience. In this sense, the use of the app seems to have strengthened the connection between school activities and domains of experience that students perceive as familiar and relevant.

> ST1: The project involved various aspects and, consequently, practical activities such as building our gnomon, the digital component, and the use of various tools. Many students felt capable; in other words, throughout the process, everyone discovered their strengths.

It appears that the variety of activities orchestrated [18] — in particular, the alternation between the use of digital applications and the construction of physical objects within a STEM-oriented framework — encouraged the participation of a large portion of the class. In the teachers' view, this diversification allowed many students to find forms of participation more closely aligned with their interests and strengths. In this way, each student was able to identify an area in which they felt particularly competent and, consequently, to perceive themselves as a valuable resource both in group work and in whole-class discussions. From the teachers' perspective, this variety of approaches to the experience therefore seems to have fostered broader and more inclusive forms of participation.

5.3. Difficulties and curriculum matter

> ST1: I have to admit that at many points I didn't feel quite up to the task. I've always needed to consult with my colleagues. [...] I found a great deal of cooperation from my colleagues; I got to know many of them on a professional level — people I hadn't known so well before — so it was certainly very important in terms of internal collaboration among the faculty as well.

> PT2: To be honest, I don't usually teach right outside, and even though the approach is, let's say, different from the one I use — which is a bit more mechanical — it was still helpful for me because I learned to teach in a different way. It was obviously difficult.

The teachers report that they did not always feel fully confident in managing practices that were largely new to them [2], and that they often turned to colleagues and didacticians for support. At the same time, the experience appears to have prompted a transformation in their teaching practices. In particular, planning outdoor activities required a high degree of flexibility, both in responding to students' contributions as they emerged and in adapting the work to unforeseen constraints, such as weather conditions.

> PT1: It was also very important to adjust our plans, because we had originally drawn up a plan that underwent a lot of changes and revisions along the way. [...] At first, we had planned to do it only in the classroom, but then the sun was out, so we went outside and took advantage of it.

> ST1: I think, based on how it went in my class, that the last part was rushed a bit, so I would devote a little more time to the final phase, which involves reflecting on the data collected. [...] I realized I had rushed through it too quickly. [...] As for feasibility, I believe it is repeatable. And if I had to think of a way to improve it, I would make it a multi-year project spanning the three years of lower secondary school.

> ST2: before ST1 talking I was thinking, I would do it again but from grade 6, and not in grade 8. And throughout the three years. That would certainly be wonderful, because, as ST1 said, it's precisely those mathematical concepts that evolve.
>
> PT2: I also agree with what my colleagues said — you should make the course a little longer. This year we started in March, so time was a bit tight toward the end, between the INVALSI[4] tests and the holidays. That part was perhaps covered a bit too hurriedly. So I'd extend it a little bit too, partly because those mathematical concepts can be covered in all grade levels, with some variation depending on the students' age.

These excerpts suggest that integrating the program into the curriculum requires, first of all, a flexible design capable of adapting both to organizational and environmental contingencies and to the actual pace of students' learning. The teachers emphasize that the work required continuous adjustments during implementation and that certain phases — particularly the final phase devoted to reflecting on the collected data — would have benefited from more time. At the same time, the reflections indicate that the program is regarded as sustainable and replicable, but only if it is situated within a broader and less episodic framework. In particular, the idea emerges of integrating it vertically into the lower secondary curriculum, beginning as early as grade 6 and developing it across multiple school years, so as to support the progressive development of the mathematical concepts involved.

6. Discussing the Outfield Education

Through the account of a teaching episode and the reflections shared by the teachers within the community of inquiry, we sought to address RQ1 by examining more deeply the different dimensions of our proposal for outdoor mathematics education, which we present at the end of this paper under the provisional name of Outfield Education (Figure 4). "Outfield" is a term derived from baseball, referring to the outermost part of the playing field, in contrast to the "infield", which contains the bases. Just as the ball is hit by the batter toward the outfield and then thrown back by the opposing team toward the infield, so too does our educational approach unfold both inside and outside the classroom, through a continuous yet carefully planned alternation intended to support specific educational aims [23]. This term appeals to us because it is a portmanteau of "outdoor education" and "field of experience".

While each framework contributes essential tools, none alone is sufficient: FoE identifies what is worth exploring mathematically but does not specify how learning unfolds across environments; OE foregrounds experiential and relational dimensions but lacks analytical precision for artefact-mediated orchestration; and IO accounts for the teacher's organization of tools but was developed primarily for classroom settings. Outfield Education names the specific configuration that emerges from their combination: one in which the field of experience itself functions as the primary learning environment, connecting formal inquiry, outdoor exploration, and students' lived worlds into a coherent educational trajectory.

By Outfield Education, we therefore mean an education that takes place out-in-the-field, that is, within the FoE itself and through its direct exploration. In this sense, the aim is not simply to propose mathematical tasks situated in a FoE while still preserving the structure of traditional classroom exercises, such as conventional word problems typically associated with the *infield*. Rather, our goal is to explore the FoE and understand how it works, using mathematics as a guiding discipline in dialogue with other relevant areas of knowledge, including not only STEM disciplines, but also more humanistic ones such as history or art.

[4]National Institute for the Evaluation of the Education and Training System

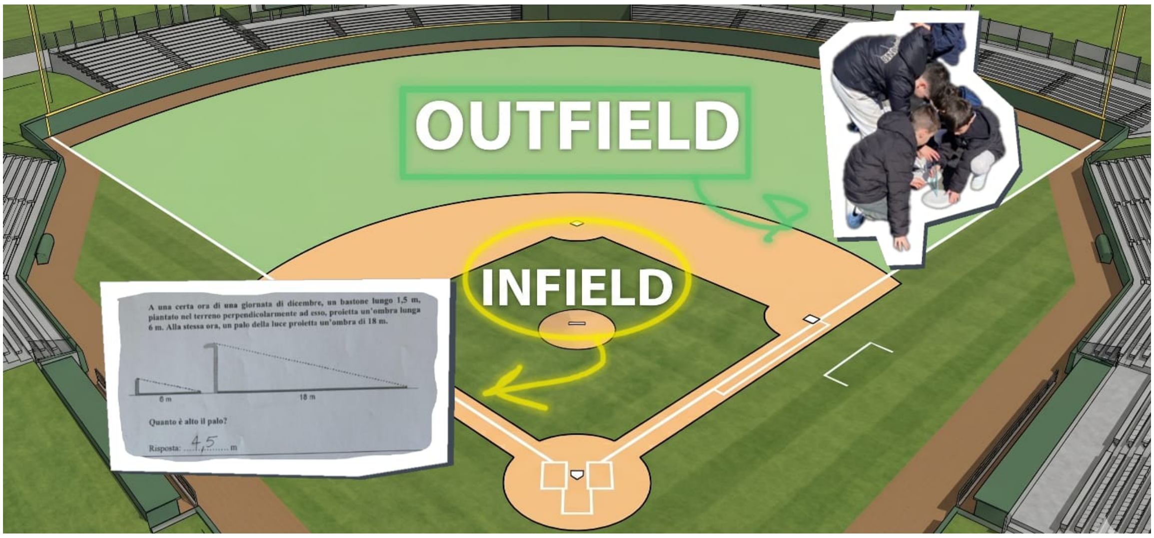

Figure 4. Image that serve as an example to show what we intend with the baseball metaphor used to mint "Outfield Education" (the background is generated through Gemini Flash 3.5 and then edited by the author)

In summarizing the different perspectives on out-of-school mathematics education, Kuş [24] identifies three categories of settings in which learning takes place: (a) everyday life environments, (b) designed learning environments, and (c) activities outside the classroom within schools. In the approach developed by the teachers, designed learning environments were certainly used — understood as informal settings intentionally structured through schedules, the presence of educators, and the provision of technologies and tools to support mathematics learning [25] — given the use of tools and technologies integrated into the learning environment [18]. However, everyday life environments, such as theme parks, and outdoor spaces adjacent to the school were also involved. Some of these choices were made precisely in order to make use of spaces already familiar to the students, where they had previously lived experiences that could intertwine with the FoE exploration activity and further enrich this body of place-related experience. From the perspective of Outfield Education, it therefore seems less important to define one specific learning environment among these three categories than to ensure that the environments serve the purpose of exploring the FoE. The FoE itself thus appears to function as the learning environment that connects the natural world we explore, the classroom space in which we engage in more formal inquiry into our understanding of the FoE, the digital world in which applications allow us to simulate that FoE, and finally the world of the personal sphere of teachers and students, which manifests itself in relationships with people and environments and in life experiences, whether actual or potential [14].

This approach to the exploration and study of the FoE is grounded precisely in relationships and collaboration. With regard to RQ2, the teachers who participated in the focus group highlighted the affordances of working through different approaches and in an outdoor context, which loosens the constraints of the classroom and fosters relationships both within and beyond it. At the same time, they also emphasized the challenges teachers encounter. Recalling the teaching episode, in the more improvisational phases in which the teacher, outdoors, is called upon to engage in didactical performance oriented toward educational aims that had not been planned for that lesson, but perhaps for subsequent ones, the lesson relies heavily on the social construction of knowledge [40]. During the outdoor sequence of the teaching episode, the teacher becomes a semiotic mediator [19]: she observes the students' work with the tools, identifies meaningful representations through which to explore a mathematical concept [36], directs the whole class's attention to that representation by asking them to reproduce it, thus working on the construction of social schemes [16], and then reflects together with them on the mathematics conveyed by that sign. We seek to explore these aspects more deeply by establishing a theoretical connection with the Instrumental Approach [16], which constitutes one of the components of our initial conceptualization of Outfield Education.

If we wish to synthesize an initial definition of what Outfield Education is, we may do so by paraphrasing Donaldson and Donaldson's [15] definition, "Outdoor education is in, about and for the outdoors", which, in our case, becomes: "Outfield Education is in, about, and through the field of experience". It refers to the way in which the teacher situates the educational process *in* the field of experience; learning is *about* the field of experience, and it is *through* education conducted within the field of experience that mathematics is learned.

7. Where are we heading now? Limitations and future perspectives

This paper presents the more theoretical contribution that emerged from a collaborative effort carried out by a very specific group of teachers who had been working for years with researchers from the University of Eastern Piedmont. Throughout the 2024/2025 academic year and into the following one, teachers and researchers worked together by revisiting previous studies on teaching mathematics through the FoE of the sun shadows [14], exploring what it means to teach through an approach that was new to them, such as Outdoor Education, and discussing for the first time the integration of digital technologies into activities and teaching. Such in-depth work was made possible by the particular nature of this group, but its structure and intensity are clearly not easily replicable.

For this reason, the present study is intended as an initial pilot and exploratory investigation, to be followed by further discussions with other groups of teachers, for whom accessible and replicable teacher education pathways will be developed on the basis of the observations gathered from this first pilot group. The aim is therefore to identify, through ongoing dialogue with the teachers who will gradually take part in these cycles, design principles [35] for teacher education in a practice such as Outfield Education, while also using this joint work and collaboration to deepen the more theoretical perspective pursued through RQ1. Since this is only an initial conceptualization, it will need to be progressively refined and clarified over time, always in collaboration with teachers. This reflects the nature of Educational Design Research [35], which offers both a practical contribution — in this case, concerning the design of a possible teacher education pathway — and a theoretical one, aimed at clarifying the teaching and learning approach on which this work is based.

Regarding RQ2, the teachers' reflections in the focus group highlighted two issues that will need to be addressed in the near future. The first concerns teacher education on this instructional approach, which differs from more traditional classroom practices while also building on them, particularly because some teachers in the community felt that they did not yet have sufficient knowledge or confidence to lead such activities. Within these teacher education pathways, Outfield Education will be studied, implemented, and further explored, since it is still at an early stage of development and requires additional refinement. Moreover, because the present research focuses primarily on teachers, the further development of Outfield Education will also require greater attention to students and their learning, which should be examined directly rather than only through teachers' accounts.

The second issue concerns curricular integration, which the teachers identified as a central challenge and one that could be addressed by distributing such activities across more than one school year. This will involve considering multi-year teacher education pathways capable of supporting teachers in the implementation of these practices, while also creating opportunities for discussion with them in order to further refine and improve teaching practice and to engage with theoretical issues and questions emerging from scientific research [41].

A further development will concern the use of video-stimulated reflection after the second year of implementation. Whereas the focus group discussed in this paper was organized around teachers' general reflections on the experience, future discussions will be grounded in selected classroom episodes. This will make it possible for teachers and didacticians to examine more closely how mathematical opportunities emerge during outdoor activity, how teachers recognize and respond to students' actions, and how artefacts and environments are orchestrated in real time.

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Conflict of interest

The authors declare that there are no conflicts of interest.

References

[1] A. Bakker, J. Cai, L. Zenger, Future themes of mathematics education research: An international survey before and during the pandemic, *Educational Studies in Mathematics*, **107**(1) (2021), 1–24, https://doi.org/10.1007/s10649-021-10049-w.

[2] C. Wolf, P. Kunz, N. Robin, Emerging themes of research into outdoor teaching in initial formal teacher training from early childhood to secondary education: A literature review, *The Journal of Environmental Education*, **53**(4) (2022), 199–220, https://doi.org/10.1080/00958964.2022.2090889.

[3] M. Rickinson, J. Dillon, K. Teamey, M. Morris, M. Y. Choi, D. Sanders, P. Benefield, *A review of research on outdoor learning*, National Foundation for Educational Research, 2004.

[4] L. Niklasson, A. Sandberg, Reflecting on field studies in teacher education: Experiences of student teachers in Sweden, *Journal of Early Childhood Teacher Education*, **33**(3) (2012), 287–299, https://doi.org/10.1080/10901027.2012.705807.

[5] R. Farné, A. Bortolotti, M. Terrusi (Eds.), *Outdoor education: Prospettive teoriche e buone pratiche*, Carocci, 2018.

[6] T. Rea, S. Waite, International perspectives on outdoor and experiential learning, *Education 3-13*, **37**(1) (2009), 1–4, https://doi.org/10.1080/03004270802291699.

[7] R. Nicol, Outdoor education: Research topic or universal value? Part two, *Journal of Adventure Education and Outdoor Learning*, **2**(2) (2002), 85–99, https://doi.org/10.1080/14729670285200201.

[8] R. Nicol, Outdoor education: Research topic or universal value? Part three, *Journal of Adventure Education & Outdoor Learning*, **3**(1) (2003), 11–27, https://doi.org/10.1080/14729670385200211.

[9] Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR), *Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione* [National curriculum guidelines for preschool and the first cycle of education], 2012.

[10] Ministero dell'Istruzione e del Merito (MIM), *Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione* [National curriculum guidelines for preschool and the first cycle of education], 2025.

[11] S. Priest, Redefining outdoor education: A matter of many relationships, *The Journal of Environmental Education*, **17**(3) (1986), 13–15, https://doi.org/10.1080/00958964.1986.9941413.

[12] R. Casi, *Informal mathematics education in museums: An exploratory study on teacher education*, Doctoral dissertation, Politecnico di Torino, 2024, https://tesidottorato.depositolegale.it/handle/20.500.14242/157530.

[13] H. Eshach, Bridging in-school and out-of-school learning: Formal, non-formal, and informal education, *Journal of Science Education and Technology*, **16** (2007), 171–190, https://doi.org/10.1007/s10956-006-9027-1.

[14] P. Boero, C. Dapueto, P. L. Ferrari, E. Ferrero, R. Garuti, E. Lemut, L. Parenti, E. Scali, Aspects of the mathematics-culture relationship in mathematics teaching-learning in compulsory school, in *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education*, PME, **1** (1995), 151–166.

[15] G. W. Donaldson, L. E. Donaldson, Outdoor education: A definition, *Journal of Health, Physical Education, Recreation*, **29**(5) (1958), 17–63, https://doi.org/10.1080/00221473.1958.10630353.

[16] P. Rabardel, *Les hommes et les technologies: Approche cognitive des instruments contemporains*, A. Colin, 1995.

[17] L. Trouche, Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations, *International Journal of Computers for Mathematical Learning*, **9**(3) (2004), 281–307, https://doi.org/10.1007/s10758-004-3468-5.

[18] P. Drijvers, S. Grauwin, L. Trouche, When bibliometrics met mathematics education research: The case of instrumental orchestration, *ZDM: Mathematics Education*, **52**(9) (2020), 1455–1469, https://doi.org/10.1007/s11858-020-01169-3.

[19] M. G. Bartolini Bussi, M. A. Mariotti, Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective, in L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed.), Routledge, (2008), 746–783.

[20] R. Noss, C. Hoyles, *Windows on mathematical meanings: Learning cultures and computers*, Kluwer Academic Publishers, 1996.

[21] P. Rabardel, R. Samurçay, From artifact to instrument-mediated learning, *New challenges to research on learning: International symposium organized by the Center for Activity Theory and Developmental Work Research*, University of Helsinki, (2001).

[22] A. Bikner-Ahsbahs, S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education*, Springer, 2014, https://doi.org/10.1007/978-3-319-05389-9.

[23] L. B. Sharp, Outside the classroom, *The Educational Forum*, **7**(4) (1943), 363.

[24] M. Kuş, Conceptualization of out-of-school mathematics education, *Journal of Education for Life*, **38**(1) (2024), 245–257, https://doi.org/10.33308/26674874.2024381644.

[25] R. Nemirovsky, M. L. Kelton, M. Civil, Toward a vibrant and socially significant informal mathematics education, in J. Cai (Ed.), *Compendium for research in mathematics education*, National Council of Teachers of Mathematics, (2017), 90–101.

[26] M. L. Kelton, Mathematics learning pathways on a school fieldtrip: Interactional practices linking school and museum activity, *Visitor Studies*, **24**(2) (2021), 220–242, https://doi.org/10.1080/10645578.2021.1939984.

[27] A. MacDonald, Young children's photographs of measurement in the home, *Early Years*, **32**(1) (2012), 71–85, https://doi.org/10.1080/09575146.2011.608651.

[28] S. F. Akkerman, A. Bakker, Crossing boundaries between school and work during apprenticeships, *Vocations and Learning*, **5** (2012), 153–173, https://doi.org/10.1007/s12186-011-9073-6.

[29] P. Gray, *Free to learn*, Basic Books, 2015.

[30] M. G. Berman, J. Jonides, S. Kaplan, The cognitive benefits of interacting with nature, *Psychological Science*, **19**(12) (2008), 1207–1212.

[31] T. Lengel, M. Kuczala, *The Kinaesthetic Classroom*, Corwin Press, Thousand Oaks, 2010.

[32] A. N. Cahyono, M. Ludwig, Teaching and learning mathematics around the city supported by the use of digital technology, *Eurasia Journal of Mathematics, Science and Technology Education*, **15**(1) (2018), em1654, https://doi.org/10.29333/ejmste/99514.

[33] B. Haas, Y. Kreis, Z. Lavicza, Integrated STEAM approach in outdoor trails with elementary school pre-service teachers, *Educational Technology & Society*, **24**(4) (2021), 205–219.

[34] B. Jaworski, Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching, *Journal of Mathematics Teacher Education*, **9**(2) (2006), 187–211, https://doi.org/10.1007/s10857-005-1223-z.

[35] S. McKenney, T. C. Reeves, *Conducting educational design research* (2nd ed.), Routledge, 2018, https://doi.org/10.4324/9781315105642.

[36] G. Vergnaud, La théorie des champs conceptuels [The theory of conceptual fields], *Recherches en Didactique des Mathématiques*, **10** (1990), 133–169.

[37] P. Mayring, Qualitative content analysis: Theoretical background and procedures, in A. Bikner-Ahsbahs, C. Knipping, N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education*, Springer, (2015), 365–380, https://doi.org/10.1007/978-94-017-9181-6.

[38] U. Kuckartz, Qualitative text analysis: A systematic approach, in G. Kaiser, N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education*, Springer, (2019), 181–197.

[39] S. Timmermans, I. Tavory, Theory construction in qualitative research: From grounded theory to abductive analysis, *Sociological Theory*, **30**(3) (2012), 167–186.

[40] L. S. Vygotsky, *Mind in society: The development of higher psychological processes*, Harvard University Press, 1978.

[41] B. Jaworski, Mathematics teaching development as a human practice: identifying and drawing the threads, *ZDM – The International Journal on Mathematics Education*, **44**(5) (2012), 613–625, https://doi.org/10.1007/s11858-012-0437-7.