



IJPAM

Italian Journal of Pure and Applied Mathematics

<https://journals.uniurb.it/index.php/ijpam>

E-ISSN 2239-0227



DOI: 10.14276/ijpam.5763

Received: 15 April 2026

Accepted: 9 June 2026

Published: 30 June 2026

Peer Review History

Single-blind peer review

Formative feedback and artificial intelligence: new perspectives for pre-service teacher education

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Abstract: This contribution investigates the role of generative Artificial Intelligence in formative feedback processes in mathematics education, with particular reference to pre-service teacher education. The study interprets an Artificial Agent, developed through ChatGPT 5.2, as a possible dynamic semiotic mediator in light of the Theory of Semiotic Mediation and the paradigm of Formative Assessment. The experimentation was carried out with 300 students enrolled in the Primary Education Sciences degree programme at the University of Bari Aldo Moro, who worked in pairs on the solution of an open-ended mathematical problem concerning the equi-extension of parts of a rectangle. The activity was structured into three phases: the production of an initial solution, interaction with the Artificial Agent through a structured prompt aimed at generating non-solution-providing feedback, and a subsequent collective discussion. The qualitative analysis of 150 protocols made it possible to observe the evolution of students' responses before and after the interaction with AI, with particular attention to solution strategies, the quality of argumentation, and processes of cognitive and metacognitive self-regulation. The results show a tendency to move from local, incomplete, or poorly argued responses towards broader, more structured, and more generalised responses, especially in cases characterised by extended dialogic interactions, reconsideration, or contestation. At the same time, some critical issues emerge, related to possible misleading feedback, overly verbose outputs, and the risk of a mechanical use of the tool. These aspects, when appropriately discussed and mediated by the teacher, may become formative opportunities for developing a

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critical and informed use of AI. The study therefore highlights that the Artificial Agent cannot replace the role of the teacher, but may constitute a significant resource for supporting revision, argumentation, and the shared construction of mathematical meanings.

2020 Mathematics Subject Classification: Primary 97C70; Secondary 97C40, 97P80.

Keywords: formative assessment; semiotic mediation; artificial intelligence; feedback; mathematics education.

1. Introduction

In the contemporary landscape of educational research, feedback has progressively become a central element in pedagogical and didactic reflection, particularly in the field of mathematics education [7]. Numerous studies highlight that timely, structured, and meaningful feedback can have a significant impact on learning processes, supporting the development of metacognitive skills, self-regulation processes, and argumentative and problem-solving abilities [11, 4].

Despite the broad consensus on the formative value of feedback, its implementation in everyday school practice still presents several critical issues. Large class sizes, organisational constraints, and limited instructional time often make it difficult to provide personalised and timely feedback capable of promoting in-depth discussions on students' productions [10]. These difficulties are particularly evident in mathematics teaching, a discipline in which feedback does not concern only the correctness of the result, but also involves the quality of the solution processes, the strategies adopted, and the arguments developed by students.

In this scenario, the spread of generative Artificial Intelligence (AI) has introduced new possibilities for transforming educational and teaching practices [6, 13]. Systems based on Large Language Models (LLMs), thanks to their capacity to support articulated and adaptive dialogic interactions, can provide personalised feedback, requests for clarification, progressive hints, and reformulations consistent with users' responses [1]. These characteristics make AI a potentially relevant tool in teaching-learning processes, moving beyond a purely technical-operational conception of technology.

The integration of AI into educational contexts, however, requires a critical and informed approach that avoids mechanical or passive uses of the tool. This implies the need to develop effective forms of AI literacy among both teachers and students, so that interaction with AI—and specifically with an Artificial Agent (AA) that is clearly defined in relation to the activity carried out—can become an opportunity for reflection, re-elaboration, and the shared construction of meanings [18].

In light of these considerations, the present study aims to investigate the role of the AA within the formative feedback process in mathematics education, interpreting it as a dynamic semiotic mediator in the light of the Theory of Semiotic Mediation [2]. From this perspective, the AA is not considered merely as a support tool, but as an artefact capable of actively participating in the construction of mathematical meanings through dialogic interactions that guide the evolution of solution strategies and argumentative processes.

The aim of this contribution is to analyse the evolution of the solutions produced by pre-service teachers in solving a mathematical problem, as well as their progressive awareness of the role of the AA as a mediator in the feedback process. In particular, the study focuses on the interactions developed between pre-service teachers and ChatGPT 5.2, with the purpose of observing how the feedback generated by the AA may influence the re-elaboration of solution strategies, the development of mathematical argumentation, and self-regulation processes.

Through this perspective, the study is positioned at the intersection of mathematics education, formative assessment, and studies on AI in education, contributing to the debate on the potential of the AA as a mediating device in processes of mathematical knowledge construction.

2. Theoretical Framework

The activity under study was designed and analysed with reference to two main theoretical frameworks: the Theory of Semiotic Mediation (TSM) [2] and the paradigm of Formative Assessment (FA) [4, 11]. The integration of these perspectives makes it possible to interpret the role of generative AI and specifically that of an AA defined within AI, not only as a technological support, but as an artefact involved in the construction of mathematical meanings and in formative feedback processes.

2.1. Theory of Semiotic Mediation and the role of AI

Developed within the Vygotskian tradition, the Theory of Semiotic Mediation considers central the semiotic relationships established among artefacts, activity, signs produced by students and the construction of mathematical knowledge [2]. From this perspective, mathematical learning emerges through processes mediated by the use of appropriately selected artefacts, employed within specific teaching practices. According to TSM, an artefact incorporates knowledge and, precisely for this reason, can offer those who use it, according to a scheme of use related to an assigned task, a way of accessing the mathematical meanings it conveys [19]. An artefact therefore assumes a semiotic mediation function when it is used by students to carry out a task and, simultaneously, by the teacher with the intention of promoting specific mathematical meanings. A central element of TSM is the concept of semiotic potential, defined as the dual link that the artefact has: on the one hand with the personal meanings produced by students during the activity, on the other hand with mathematical meanings. Through the teacher's mediation and moderation actions, particularly in the phases of collective discussion [3], individual signs can progressively evolve into shared and mathematically recognised signs. In the present contribution, generative AI makes it possible to characterise an AA interpreted as an artefact endowed with a specific semiotic potential. Unlike traditional manipulative or digital artefacts, the AA is characterised by the ability to support dynamic dialogic interactions, progressively adapting its responses to users' productions. This characteristic makes the AA a particularly relevant semiotic mediator, since it is capable of accompanying the student in the explicit formulation of their reasoning, in the reformulation of solution strategies and in the progressive construction of mathematical meanings. Dialogic interaction with the AA can in fact foster processes of clarification, argumentation and revision, stimulating the student to reflect on their own procedures and choices. From this perspective, the AA does not merely provide information or corrections, but actively participates in the construction of mathematical meaning, orienting students' actions and productions.

2.2. Formative Assessment and Feedback in Mathematical Learning Processes

Alongside the perspective of semiotic mediation, the present study is grounded in the paradigm of Formative Assessment, understood as a set of practices aimed at collecting and interpreting information on students' learning processes in order to guide subsequent teaching decisions [4, 5, 8].

Within Formative Assessment, feedback assumes a central role. Previous studies in pre-service teacher education have highlighted how feedback, when embedded in structured formative assessment paths, can support the development of communicative skills, reflective processes and awareness of one's own mathematical work [9]. In the field of mathematics, it cannot be limited to the identification of errors or to the validation of the correct result, but must support the explicit formulation of the strategies adopted, highlight possible conceptual difficulties and promote processes of revision and self-regulation. Effective feedback in fact contributes to the development of metacognitive awareness and supports the construction of argumentative and problem-solving competences [11].

According to Black and Wiliam [4], Formative Assessment takes place when the evidence col-

lected during the activity is used by teachers, students or peers to make teaching decisions that are more effective than those that would have been made in the absence of such information. Wiliam and Thompson [20] identify five key strategies of Formative Assessment: clarifying learning goals and success criteria; promoting effective discussions; providing feedback that guides improvement; activating students as instructional resources for one another; supporting processes of self-regulation and responsibility.

Within this framework, peer feedback assumes particular relevance, since it fosters processes of comparison, negotiation and shared revision of mathematical productions. In order for feedback to be genuinely formative, it is necessary that it enables students to recognise critical issues, gaps or inconsistencies and to identify possible strategies for improvement [17].

Hattie and Timperley [11] distinguish four levels of feedback: feedback on the task, related to the correctness of the activity carried out; feedback on the process, referring to the strategies used; self-regulation feedback, oriented towards the student's ability to monitor and control their own learning; feedback on the self, related to personal and emotional aspects. In the present study, particular attention is devoted to feedback on the process and on self-regulation, considered fundamental for supporting the evolution of the solution strategies and argumentative levels of pre-service teachers.

From this perspective, Formative Assessment can be interpreted as a mediating device, through which the teacher, and, in the present study, also the AA, contributes to the construction of meaning from the signs produced by students, accompanying them in the processes of re-elaboration, revision and construction of mathematical thinking. In their central role in designing the activity, defining the tasks, choosing the ways in which the AA is used and structuring the collective discussions aimed at the negotiation of meanings, the teacher ensures that interaction with the AA can effectively promote an evolution of mathematical meanings.

In light of these theoretical references, the present work aims to analyse to what extent interaction with an AA, interpreted as a dynamic semiotic mediator within the formative feedback process, can affect the re-elaboration of solutions to a mathematical problem and the evolution of the argumentative strategies of pre-service teachers. In particular, the research questions that guided the study are:

RQ1. How does dialogic interaction with an AA, interpreted as an artefact and dynamic semiotic mediator within a formative feedback process, affect the evolution of the solution strategies and argumentative levels adopted by pre-service teachers in solving an open-ended mathematical problem?

RQ2. What processes of cognitive and metacognitive self-regulation emerge in the interactions between pre-service teachers and the AA during the revision of mathematical solutions?

3. Methodology

3.1. Choice of the problem

The experimentation was conducted with students enrolled in the Primary Education Sciences degree programme at the University of Bari, during the Mathematics Education course. The sample consisted of a total of 300 pre-service teachers. The activity was carried out in pairs, this choice allowed peer comparison and fostered the sharing of solution strategies.

With regard to the assigned task, the following open-ended problem was chosen [16]: *“Two brothers inherit a rectangular plot of land. In order to divide it into two parts of the same area, an acquaintance suggests that they place a pole at any point on the land and connect it to the four small posts fixed at the four vertices of the rectangular plot. One of the brothers will take the part coloured grey in the drawing, the other will take the remaining part. Are the two parts really equal? Justify your reasoning.”*

This choice is related to the plurality of applicable solution strategies: the separate or

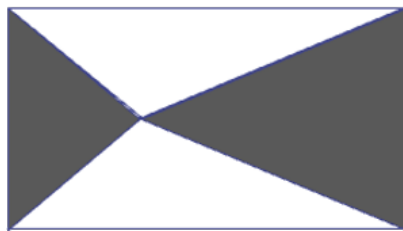


Figure 1. The image of the land

combined application of algebraic, geometric and empirical strategies makes it possible to propose, or re-propose, the problem at different school levels. The importance of these problems lies in the possibility of stimulating logical-critical thinking, argumentation and creativity in solution methods. Unlike closed problems, this type of problem allows teachers to analyse and understand different mathematical competences, different approaches to problem solving and possible misconceptions, information that is useful for creating personalised pathways to guide the teaching-learning process from a formative perspective.

3.2. Structure of the experimentation

The phases of the experimentation are divided as follows:

- Phase 1: Problem solving: the pairs of students produced a first solution to the problem and entered their answers in a Google Form. The form was prepared in order to keep track of the different strategies adopted and arguments made explicit.
- Phase 2: Interaction with AI: in this phase, students were given the opportunity to interact with ChatGPT 5.2. The interaction began with the insertion of a prompt structured in advance, through which it was possible to instruct the Chatbot to provide indications that did not solve the problem, but acted as catalysts for critical thinking.

The choice of a prompt structured in advance is related to the importance of providing the chatbot with the necessary information to construct a high-quality and meaningful interaction [12].

The structured prompt for Primary Education Sciences students is:

I am a Primary Education Sciences student, and I have solved the following mathematics problem. I would like you to help me reflect on the steps of my solution without providing me with answers, but by giving me formative feedback. The text of the problem is “Two brothers inherit a rectangular plot of land. In order to divide it into two parts of the same area, an acquaintance suggests that they place a pole at any point on the land and connect it to the four small posts fixed at the four vertices of the rectangular plot. One of the brothers will take the part coloured grey in the drawing, the other will take the remaining part.

Are the two parts really equal? Justify your reasoning. My solution is -insert your own solution with the image-.

Following the exchange with ChatGPT 5.2, the students entered both the interactions with the chatbot and the reformulated answers in a further Google Form.

In the analysis of the responses, it was possible to highlight the evolution of the solutions, derived from the dialogic interactions with the AA: the continuous reciprocal feedback was a crucial element in the transformation of the initial answers.

- Phase 3: Collective discussion: the solutions returned by the students were then the object of collective discussion [3]. This choice is also consistent with previous research on Mathematical Discussion in pre-service teacher education, which has shown the importance of learning in and from practice through digitally mediated discussion environments

[15]. In this discussion, issues of both a mathematical nature and concerning the unprecedented role of the AA were addressed. The teacher, in this phase, played a central role as a bridge for the mathematical institutionalisation of the contents and for the critical comparison on the new practices in which the AA intervened.

4. Analysis of the Interaction with the AA: Two Case Studies

The analysed sample consists of 150 protocols (300 students divided into pairs), structured as follows: initial answer (before the introduction of the AA), interaction with the AA and reformulated answer (after the use of the AA). In the following analysis, the solutions to the problem and the different interactions with the AA were classified. The types of answers identified are the following:

- Local answer: the answer is correct but refers only to one or more particular cases, without grasping or making explicit the general structure of the problem.
- Incomplete answer: the answer provides a correct result, but without an explicit argument or with omitted logical steps.
- Broad answer: the answer is correct, argued and formulates a general solution and includes multiple solution strategies.
- Incorrect answer: the answer contains conceptual or procedural errors that lead to a false conclusion.

Data analysis was conducted according to a qualitative exploratory approach, through the classification of the protocols and interactions on the basis of categories consistent with the theoretical framework of reference. As recalled in the theoretical framework, the AA is an active mediator in the construction of meaning: this construction takes place through feedback that provides continuous support to the user. This mode of interaction, guided by the teacher's a priori design, makes it possible to frame ChatGPT, in version 5.2, as a possible dynamic semiotic mediator, capable of transposing knowledge and orienting the learning of the interlocutor. From the analysis of the data, multiple modes of interaction with the AA emerge: the types of conversations observed present differences in language, in the quality of the dialogue and in the breadth of the exchanges. The effects of the suggestions provided by the chatbot produced different reactions both at the cognitive level and at the metacognitive level. Indeed, during the dialogue phase with the AA, students were able to ask for in-depth explanations of the mathematical concepts involved and for a revision of their own solution processes useful for the reformulation of the solutions. Following a more or less argued reformulation of the solution strategy, the answers were placed at a new level with respect to the initial classification. In addition to the classification of the answers mentioned above, it was possible to highlight the different modes of interaction between the AA and the user, grouped into five operational categories:

- Rapid interaction: sequence of 1–2 conversational turns, oriented exclusively towards verifying the correctness of the answer, without requests for clarification or in-depth analysis.
- Minimal interaction: substantially passive acceptance of the feedback generated by the AA, in the absence of reformulations, explanatory requests or revision of the initial solution.
- Broad interaction: structured and prolonged dialogue (several turns), aimed at conceptual exploration, understanding errors and analysing solution strategies.
- Rethinking interaction: interactive process that leads to an explicit modification of the initial solution, with reorganisation of the procedure or reformulation of the answer.
- Contestation interaction: interaction characterised by the critical questioning of the AA's feedback, through arguments, counterexamples or requests for justification.

The analysis of two case studies of protocols produced by pre-service teachers is reported below.

4.1. The first case

The protocol presented in this first case was produced by a pair of Primary Education Sciences students during the Mathematics Education course.

The initial answer was classified as local and incomplete, since the invariance of area was only partially recognised, with reference to a particular position of the point, and the strategy presented was empirical-visual in nature.

The students' initial answer is reported below:

We measured the land with a ruler and reproduced it on paper with measurements of 10 cm and 5.5 cm, also drawing the 4 triangles inside. Then we cut out the various parts, thus obtaining the 4 separate triangles. By joining the two pairs of triangles, we obtained two equal figures. Therefore yes, the two parts are really equal. In addition, we noticed that by joining the two white triangles and the two black triangles, the resulting figure is a rhombus, whose diagonals are the base and the height of the original rectangle.



Figure 2. Examples of procedures

From the first lines of the solution, it emerges that the students used measurement to reproduce the figure on paper, taking the measurements directly from the screen of the pc used during the activity. The use of measurement in this way suggests the need to make concrete and to “mathematise” the proposed problematic situation through numbers. Subsequently, they cut out and superimposed the different parts of the rectangle, trying to demonstrate the invariance of area through a visual proof. This strategy inevitably entails a lack of abstraction and generalisation, which makes the proof provided not sufficiently complete.

After the students entered the initial answer in ChatGPT, the first feedback received was the following:

Your work shows:

- *good modelling ability;*
- *conscious use of concrete representations;*
- *active exploration of the geometric figure;*
- *first steps towards generalisation.*

The reflections I suggested could help you make your argument even stronger and better understand which mathematical ideas are involved. If you want, we can also explore together one of the steps to clarify how to transform your intuition into a more general justification

The analysis shows that the AA, thanks to the indications included in the initial prompt, does not directly provide the solution but invites students to review, re-elaborate and deepen their argumentation. Moreover, in the continuation of the conversation, reported in part below, the chatbot proposes guiding questions and suggestions for alternative solution strategies

in order to improve the deep understanding of the problem and allow for more structured argumentation.

Students: *From a theoretical point of view, what suggestions could you provide us?*

ChatGPT: *Here are some theoretical suggestions that can help you strengthen your reasoning without replacing your empirical path with a “ready-made” solution. The aim is to show you which mathematical ideas can support and make more general what you have already discovered through practical work [...]*

Students: *So in your opinion what is the step that is missing in our reasoning and that we should therefore add?*

ChatGPT: *I would say that the step that is really missing in your reasoning, the one that would allow you to transform an excellent empirical verification into a theoretical justification, is to make explicit why the two pairs of triangles necessarily have the same area independently of the position of the chosen point. In other words: You need to theoretically justify the equality of the areas of the two pairs of triangles, without relying only on the physical superimposition obtained through cutting out.*

The exchange highlights that the students do not settle for the first feedback received from the AA, but ask for further clarification in order to improve their answer.

With reference to the previous classification, we interpreted this interaction as broad and rethinking.

Indeed, in the third phase, that is, the reformulation of the answer, an understanding of the invariance of area independently of the experimental verification emerges, as can be inferred from the following:

[...] To this concrete verification we added a geometric reflection in order to understand why this equality of area does not depend on the specific drawing but always holds, whatever point is chosen inside the rectangle. The opposite sides of a rectangle are parallel and have the same length: this means that, if we consider for example the upper side and the lower side, the two triangles that have them as their bases have as their respective heights two distances whose sum always gives the total height of the rectangle. Since the area of a triangle is always half the product of base and height, the sum of the areas of these two triangles is exactly half of the area of the whole rectangle. The same reasoning can be applied to the other pair of opposite sides, again obtaining half of the total area. Therefore, both by observing the cut-outs and recomposing them, and by reasoning geometrically, we concluded that the two parts obtained by connecting the internal point to the vertices really have the same area, independently of where the chosen point is located.

The reformulated answer highlights a relevant change in the nature of the argumentation: from empirical verification through measurement, cutting out and recomposition, the students move to a justification based on the geometric properties of the rectangle and the triangles. This transition emerges when the students state that they added to the “concrete verification” a “geometric reflection” in order to explain why the equality of area does not depend on the specific drawing, but holds for any position of the internal point. The reference to the opposite sides of the rectangle, parallel and congruent, allows them to shift their attention from the measured figure to the geometric structure of the problem. In particular, the sentence “Since the area of a triangle is always half the product of base and height, the sum of the areas of these two triangles is exactly half of the area of the whole rectangle” shows the achievement of a more structured and complete level of argumentation. The students no longer rely only on visual superimposition, but use the relationship between bases, heights and areas to justify invariance. Moreover, the statements “The same reasoning can be applied to the other pair of opposite sides” and “independently of where the chosen point is located” indicate an evolution of mathematical thinking towards generalisation. Therefore, there is a transition from a local and incomplete answer to a complete answer, based on geometric properties and on the

understanding of area invariance.

Focusing on the role of the AA, several formative aspects can be recognised, such as the dialogic and encouraging climate, the stimulus to reflect on and revise the reasoning. From what emerged, the choice to analyse this specific case of interaction derives from the possibility of clearly observing the constructive and critical use of the AA, which allowed a semiotic evolution of the mathematical meanings related to the assigned problem.

From the perspective of TSM, the students' initial empirical and visual productions can be interpreted as personal signs emerging from the use of concrete actions such as measuring, cutting and recomposing. Through the dialogic feedback provided by the AA and the subsequent teacher-led collective discussion, these signs progressively evolved towards more explicit mathematical signs, grounded in the properties of the rectangle, the relationship between bases and heights, and the invariance of area.

4.2. The second case

The second protocol analysed falls within the category of an incomplete and broad answer. The pair of students involved provided the following initial solution:

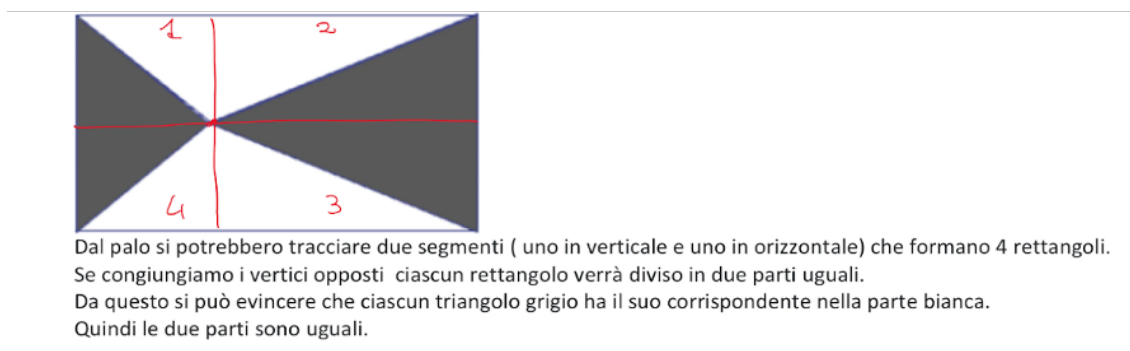


Figure 3. Initial solution produced by a pair of pre-service teachers.

Translation of the students' written response: “From the pole, it would be possible to draw two segments, one vertical and one horizontal, which form 4 rectangles. If we join the opposite vertices, each rectangle will be divided into two equal parts. From this, it can be inferred that each grey triangle has its corresponding part in the white region. Therefore, the two parts are equal.”

From the image, it can be observed that the strategy used includes both geometric arguments, deducible from the presence of words such as “segments”, “rectangles”, “vertices”, and visual ones: the demonstration of equal area is accompanied by the representation of the figure in which there is the subdivision and correspondence of the parts coloured grey and white. Moreover, the particular position of the point leads to the demonstration of a specific case and therefore indicates a lack of generality. The proposed argument is rich in implicit elements, and despite the presence of geometric terms, it does not present a rigorous theoretical basis. To this formulation the chatbot provided the following feedback: *Good job, this is a nice attempt, it is clear that you have already intuited the right direction. I will give you specific feedback on the steps, without “spoiling” the final answer, with suggestions to make the reasoning more precise and rigorous [...] 2. Correct the description of the figure Instead of speaking of “4 rectangles”, it is more accurate to say that by connecting the internal point to the four vertices, 4 triangles are obtained. This clarifies the nature of the areas that you need to compare.* It can be noted that ChatGPT's feedback is not entirely exhaustive or clear, but it provides implicit suggestions about the geometric figures to be considered, triangles instead of rectangles, and it does not describe with precision the logical steps necessary for the students' understanding. Indeed, they ask for further explanations: Students: *Can you explain point 2 to me with a*

drawing? ChatGPT:

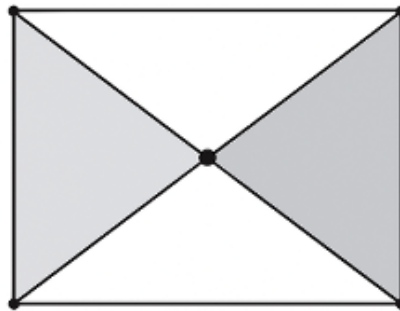


Figure 4. Visual representation provided by ChatGPT in response to the students' request for clarification.

The AI feedback, in this case, is misleading for the user: the figure provided shows the pole at the centre of the rectangle, thus bringing the problem back to a particular case.

The possibility of incorrect suggestions from ChatGPT constitutes a real risk when the AA is used mechanically [14]. Moreover, the subsequent dialogue was characterised by very extensive feedback, for which the students explicitly asked for a reduction and simplification of the information provided.

From our point of view, the presence of hallucinations, large outputs and the various possible critical issues becomes a potential resource in the phase of collective discussion, since reflection can stimulate critical thinking and a critical use of the AA. Indeed, the students involved in this conversation were not diverted by this error; on the contrary, they took the opportunity to ask for further explanations, creating an interaction of contestation, which step by step led to the reformulation of the answer:

Let $ABCD$ be the rectangle and let P be any point inside it. By connecting P to the four vertices, the rectangle is divided into four triangles. In the drawing, two triangles are coloured grey and form the part assigned to one brother. To compare the areas, we observe that each triangle has as its base one side of the rectangle and as its height the distance of point P from that side. Let h_1 and h_2 be the heights of the two grey triangles with respect to their bases, which are two opposite sides of length b . Since the heights add up to the whole distance between the opposite sides, we have:

$$h_1 + h_2 = H,$$

where H is the total height of the rectangle with respect to those sides. The total area of the two grey triangles is therefore:

$$A_{grey} = \frac{b \cdot h_1}{2} + \frac{b \cdot h_2}{2} = \frac{b \cdot (h_1 + h_2)}{2} = \frac{b \cdot H}{2}.$$

Similarly, the other two non-coloured triangles together have the same area. The two parts are therefore always equal, independently of the position of point P , because each includes two triangles whose sum of the areas is half the area of the rectangle.

The final answer to the problem therefore becomes complete and broad: it presents a dual algebraic and geometric strategy expressed through multiple argumentative registers.

The possibility of showing the critical issues, as well as the potentialities of this tool, motivates the choice of this protocol. It provides several points for reflection regarding the conscious

use of AI in teaching-learning processes, actively involving pre-service teachers in effective training.

5. Conclusions

The analysis of the protocols reveals a tendency to move from local and incomplete answers to broader and more argued answers following the interaction with the AA, particularly in cases characterised by broad and rethinking interactions.

The present work explored the role of the AA as a dynamic mediator in the feedback process in mathematics, in particular the effects on the solutions to the problem proposed to the students were analysed.

With structured didactic design, it is possible to include the AA as a significant support in revision and argumentation processes. The results seem to highlight that AI cannot replace the teacher; on the contrary, it requires a critical and situated use, which must be designed and planned in advance by the teacher.

In this sense, the activity also functioned as an opportunity for developing AI literacy among pre-service teachers. Rather than using the AA as a source of ready-made answers, students were guided to interrogate, evaluate and, when necessary, contest the feedback received, thus developing a more critical awareness of the educational use of AI.

Thanks to the methodology identified and to the structuring of the phases of the experimentation, as well as to the decision to instruct the chatbot with a specific prompt, it was possible to observe the evolution of the answers. The comparison between the productions before the AA and after the AA made it possible to observe changes in solution strategies and the evolution of the levels of argumentation.

The choice of the open-ended problem seems to be essential for highlighting the different solution strategies, bringing out the main difficulties encountered and making explicit the effect of feedback. Indeed, from the analysis of the protocols, the transition from local and incomplete answers to broad answers emerged. Despite the positive evidence presented, AI showed several critical issues such as misleading suggestions and verbose outputs. These critical issues provided material for reflection in the third phase of the experimentation, namely the collective discussion, in which several beliefs concerning a mechanical and little aware use of AI emerged. Through the alternative methodology used in the experimentation, in which the AA assumed a role different from the usual one, the students modified their schemes of use of this tool, recognising its scaffolding potential when used analytically. Contestation interactions therefore assumed a particularly relevant role, since they activate forms of control and regulation of the process.

In conclusion, the integration of the AA into the feedback process can represent a significant resource for mathematics education. It can support problem solving and foster the development of cognitive and metacognitive autonomy if designed and used critically by all the actors involved in the didactic activity.

Conflict of interest

The authors declare that there are no conflicts of interest.

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