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Children as mediators of home-school continuity in early mathematics: the case of Betta-the-Bee

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Abstract: This paper examines how a mathematically rich card game, *Betta-the-Bee*, supported home-school continuity in early mathematics when it was first introduced in kindergarten and later taken home by children. The study was conducted in a public kindergarten in Turin, Italy, and adopted an exploratory case study design. Data came from three parent questionnaires, an online focus group with three parents, and a supporting interview with two teachers. The analysis focused on parents' initial recognition of mathematics and mathematical play, on the home re-enactment of the game, and on the forms of continuity that emerged through that process. The study contributes to research on home-school continuity in mathematics by identifying children's mediation of a rule-governed mathematical practice as a plausible mechanism through which continuity may be sustained across settings.

2020 Mathematics Subject Classification: Primary 35B65; Secondary 35J70, 35R09.
Keywords: home-school continuity; early childhood mathematics education; educationally rich mathematical games; question formulation.

1. Introduction

As Cahoon et al. [1] recall, a substantial body of research has shown that mathematical competencies developed in the first years of life are related to later educational trajectories and broader life outcomes. Beyond school, children encounter mathematics in everyday routines, in interactions with objects and people, and in the informal situations through which children begin to organise quantity, space, relations, and patterns. For this reason, any account of early mathematics education that focuses exclusively on classroom instruction risks overlooking a decisive part of children's experience. As Phillipson et al. [2] argue, families are children's first mathematics educators, even when the mathematical character of everyday interaction is not fully explicit to the adults involved.

This broad recognition, however, does not resolve a central educational problem. If children's mathematical learning develops across home and school, it is necessary to consider how

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continuity between these contexts can be built. The literature on home-school partnerships in mathematics suggests that this is far from straightforward. Muir [3], for instance, shows that parents are often supportive of school mathematics initiatives while remaining uncertain about what happens in mathematics lessons and about how to engage productively with their children's learning. In a similar vein, Phillipson et al. [2] argue that children's learning can be predicted by the quality of the potential engagement and the extent to which families are supported in recognising meaningful opportunities for mathematical activity. The problem, then, lies not only in participation, but also in fostering intelligibility, mediation, and shared practice.

Several studies have addressed this issue by trying to connect home and school through games, books, resource bags, workshops, or forms of guided family participation. In the chapter "Using Mathematics to Forge Connections Between Home and School", Muir [3] describes initiatives designed to improve parental understanding of contemporary mathematical practices and to foster more regular numeracy-related activity at home. Streit-Lehmann [4], in turn, presents a kindergarten-based project in which families borrowed games and picture books with mathematical content to use at home, showing that structured cooperation between kindergarten and families can have positive effects on children's mathematical learning. Gervasoni [5] reports similarly that when educators and families are brought together around mathematical noticing, discussion, and play, parents can become more aware of the mathematical potential of everyday activities. More recently, Young et al. [6] showed that adding a family mathematics component to a naturalistic, game-based classroom intervention produced positive effects on preschoolers' mathematics knowledge, suggesting that the coordination of mathematics learning opportunities across school and home can add value to classroom-based work. This literature demonstrates that continuity between settings can be cultivated, but it leaves open a question that is especially relevant for early childhood: through what mechanism does that continuity become active within family interaction?

That question is particularly pressing in early mathematics, where both the content and the form of mathematical activity can be difficult for adults to identify. Cahoon et al. [1] insist on the value of qualitative approaches because parents' accounts of mathematics at home are shaped by cultural expectations, everyday routines, and variable conceptions of what counts as mathematics in the first place. Their study also shows that home mathematics environments are structured by parents' views, expectations, and attitudes, and by the extent to which school-related influences become woven into daily life. For these reasons, it seems insufficient to address the issue of continuity simply by providing materials that can be given to children to take home. Instead, opportunities for interaction within the home environment need to be structured – potentially involving materials to take home – which can help to refocus the joint attention of adults and children.

Kinnear et al. [7] emphasise that early mathematics learning gains educational force when children encounter meaningful, purposeful, and connected mathematical experiences rather than fragmented exercises or decontextualised routines. In the same volume, Muir [3] argues that parents need access not only to activities, but also to the mathematical practices that give those activities educational value. This perspective is especially relevant to games: a game can be mathematically productive not simply because it contains numbers or countable elements, but because its rules organise attention, decision-making, comparison, and inference in mathematically consequential ways. From this point of view, some games may be especially well suited to crossing the boundary between school and home, if they are first made intelligible within classroom practice.

The present paper examines one such case, focusing on the card game named *Betta-the-Bee*. Betta-the-Bee is a card game developed for players aged from 4 to 7 by a mixed research group composed by researchers in mathematics education, kindergarten and primary school teachers, and prospective teachers from the University of Torino [8]. In the game, one player hides Betta under a card and the other player must identify the correct card by asking yes/no questions. The game is therefore structured around the recognition of attributes, the formulation of questions, the interpretation of answers, and the elimination of possibilities. In the study discussed here, Betta-the-Bee was first introduced in kindergarten[2] and then taken home during the Christmas break, with children asked to teach the game to parents or other adults. This design makes it possible to examine continuity in a particularly sharp form: the same game appears in two settings, but it reaches the second setting through children who have already encountered and learned it at school. The study therefore addresses learning beyond the classroom by focusing on the movement of a mathematical practice from kindergarten to the home environment during a period of school interruption.

2. Theoretical background

In early mathematics education, continuity between home and school is often treated as desirable, but it remains difficult to establish in substantive ways. The more demanding question concerns the circulation of mathematical practices across settings: how mathematical activity becomes recognisable, meaningful, and worth engaging with both at school and at home.

This is crucial in the early years, when children's mathematical learning begins well before formal schooling and develops through family routines, play, and everyday interaction. While kindergarten mathematics increasingly takes shape through pedagogically structured activities, home mathematics are generally spontaneous and rely just on parents' views, expectations and attitudes. When home and school experiences remain weakly connected, school mathematics risks appearing bounded within the classroom, while parents may struggle to identify what counts as mathematics in early childhood or how it is being approached.

Research on home-school connections in mathematics has shown that the issue is not a lack of parental interest. Parents often value mathematics and want to support their children, but they may have only partial access to the forms of activity and discourse through which mathematics is organised at school. Continuity therefore depends on the development of shared interpretive ground through which school practice can become more visible and understandable in family life. Muir [3] makes this point clearly in showing that home-school connections become more substantial when parents are helped to understand what children are doing mathematically and why it matters. Similar concerns appear in the volume *Engaging Families as Children's First Mathematics Educators*, where the emphasis falls on awareness, shared language, and meaningful engagement rather than on participation in the abstract [5, 2].

A second relevant strand of literature concerns parents' views of early mathematics and the home mathematics environment. The home mathematics environment has been described as a combination of activities, expectations, interactions, and resources through which children encounter mathematics outside school. Cahoon et al. [1] show why this environment should not be reduced to a checklist of activities. Their qualitative study highlights the cultural and contextual character of the home mathematics environment and shows that parents differ not only in what they do, but also in how they recognise and describe mathematics. A parent may count, compare, sort, or play rule-based games with a child without naming any of this as mathematics. Conversely, mathematics may be understood in narrowly school-based terms and therefore recognised mainly when numbers, calculation, or overt instruction are present. In this respect, the home mathematics environment is shaped as much by recognition as by activity. Cahoon et al. [1] make this explicit when they argue for qualitative approaches capable

[2]In the Italian context, *scuola dell'infanzia* refers to the educational segment attended by children aged 3 to 6. We are aware that, in different cultural and educational contexts, the terms *school* and *kindergarten* may refer to distinct educational levels. In this paper, however, we use *school* in a broad sense to include kindergarten, in line with the Italian institutional and cultural context in which the study was conducted.

of capturing how parents talk about mathematics, what they notice, and how school-related practices come into daily life. Similar concern emerges in the family-engagement literature. Several contributions in *Engaging Families as Children's First Mathematics Educators* suggest that many parents are surprised to discover how much mathematics is already present in ordinary play and interaction. One educational task then is to help families recognise a broader range of activities as mathematics education, by understanding what counts as mathematics in their eyes [5].

The third conceptual strand concerns educationally rich mathematical games. Russo et al. [9] use the notion of educationally rich mathematical games to distinguish mathematically substantial games from activities that are merely enjoyable or only superficially educational. Their key point is that mathematics must be internal to the activity itself. A game is educationally rich when the player's decisions, actions, and interpretations are organised by mathematically meaningful relations. This is consistent with research on family mathematics showing that games can provide an approachable context for parent–child interaction around mathematical ideas, supporting co-play, mathematical talk, and repeated engagement at home [6]. This view resonates with Kinnear and Wittmann's [10] argument for mathematically founded early childhood activities. Their concern is not to oppose play and learning, but to distinguish activities structured on mathematics from those in which mathematics is little more than decoration. From this perspective, the rules of a game are part of its mathematical substance: they shape what counts as a useful move, a relevant distinction, or an effective strategy. If the mathematical value of a game depends mainly on what an adult later chooses to extract from it, then its educational force may weaken once it moves from school to home. If, instead, the mathematical work is embedded in the activity itself, then the game has a stronger chance of carrying a recognisable practice across settings. We believe that this latter is the case of the Betta-the-Bee game that we briefly present in the following subsection.

3. The mathematical specificity of the Betta-the-Bee game

The game Betta-the-Bee (*Betta l'apetta* in Italian), which can be downloaded free of charge under a Creative Commons licence from the website www.dfedidamath.unito.it, consists of 24 flower cards and one small "Betta" card (Figure 1). At the beginning of the game, the flower cards are placed on a table with the illustrated side facing upwards, and one player (the hiding player), out of sight of the opponent (the seeking player), chooses one card under which to hide the Betta card. By observing the cards displayed on the table, the seeking player must formulate a question about one of the variables represented on the cards, namely the number of petals, the colour of the pot, the position or number of butterflies, the position of the caterpillar, or the position of the leaf (Figure 1 left). The hiding player may respond only "Yes" or "No". On the basis of this answer, the seeking player discards from the table all the cards under which Betta could not possibly be hidden. For instance, if the seeking player asks, "Is Betta-the-Bee hidden under a red pot?" and the hiding player answers "No", all the cards showing a red pot are discarded from the table. Conversely, if the answer is "Yes", all the cards with a red pot remain on the table, while those with a blue or pink pot are discarded. During play, children take turns assuming the roles of hiding player and seeking player. When the game is played in a small group, one child may take on the role of hiding player while a group of children collectively assumes the role of seeking player; within this group, each child takes turns asking a question, and the children work together to discard the cards.

The game requires players to draw on linguistic, numerical, and spatial abilities in each of the three phases that make up a turn of play: formulating questions, answering questions, and discarding cards. As indicated in the National Guidelines for the Italian curriculum in kindergarten, primary and middle school [11], these abilities are among both the expected competences to be developed by the end of preschool and the learning objectives established

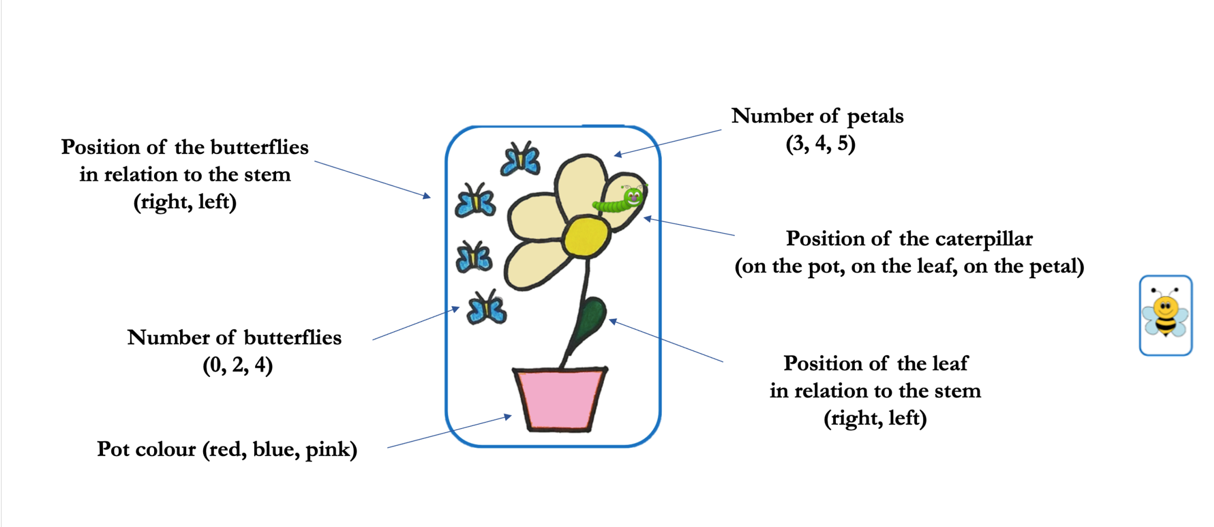

Figure 1. One of the cards of the Betta-the-Bee game with a description of variables and modalities (left); Betta-the-Bee card (right)

for the end of the third year of primary school. In our view, however, the most educationally significant aspect of the game lies in the logical dimensions it can foster from the preschool years onward.

More specifically, the different phases of the game promote forms of reasoning based on the law of excluded middle, the use of negation, material implication, and reasoning by cases (for a more detailed discussion, see [12]). The importance of fostering logical reasoning in early childhood has been highlighted by several studies, including Núñez [13], which points to a causal relationship between logical abilities and mathematical achievement in six-year-old children. Because these logical aspects are embedded within a playful and stimulating context, children can become familiar with them without their being introduced in a formal manner. We referred to this mathematics as Mathematics-in-the-Game (MiG), to distinguish it from the another type of mathematics embedded in Betta-the-Bee: the Mathematics-in-the-Deck (MiD) which refers to the mathematical potential embedded in the design of the cards themselves, independently of the reasoning strategies activated during play. In Betta-the-Bee, MiD includes basic early mathematical content such as number sense and spatial awareness, represented through variables such as the number of petals, the presence or number of butterflies, and the positions of the leaf and the caterpillar. From a mathematics education perspective, however, its main significance lies not only in these specific contents, but also in the fact that the deck presents children with multiple variables at the same time, each one represented with different possible modalities, inviting them to notice, distinguish, and attend to relevant features while playing. In this sense, the mathematical richness of the deck resides in the structured set of attributes through which children engage with comparison, categorisation, and observation in a playful context.

The MiD-MiG distinction serves an analytical purpose in this paper: MiD is certainly the most visible mathematical aspect, even to parents, but it is the discovery of MiG – through the dynamics of play supported by the rules – that enables a structured setting to be recreated at home, one that focuses adults' attention on meaningful mathematical practices.

4. Method

The study adopted an exploratory case study design, appropriate for examining a bounded educational experience in depth and for generating analytic insight [14]. In particular, it examined what happened when Betta-the-Bee, first experienced at school, was later taken into the home by children. The study was guided by three research questions:

1. How did parents initially recognise mathematics and mathematical play in kindergarten and at home?
2. How was Betta-the-Bee re-enacted in family interaction, and what role did children play in mediating that re-enactment?
3. How did the home experience with Betta-the-Bee broaden what parents recognised as mathematical in the game?

These questions called for a design able to hold together different forms of evidence: parents' initial views of mathematics, the place of games in home life before the intervention, parents' responses after the home experience with Betta-the-Bee, and more detailed accounts of how the game was introduced and played in family settings. For this reason, the study drew on multiple data sources, each serving a complementary analytic function within the exploratory case study design.

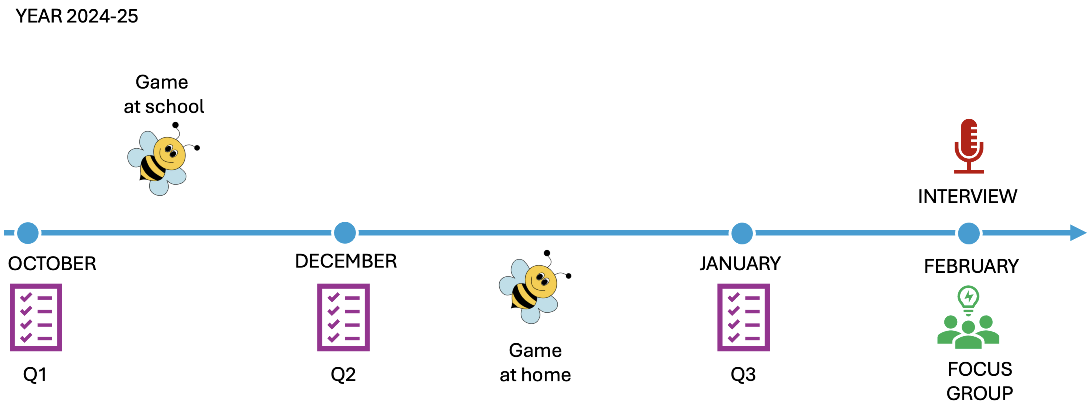

Figure 2. Timeline of the research

The study was carried out in a public kindergarten in Turin between October 2024 and February 2025. The wider school context included six sections. Within this setting, two sections were selected for the pathway centred on Betta-the-Bee. Two teachers were involved, and the core group for the home-school phase of the study consisted of 18 five-year-old children and their parents. In this setting, we collected the following data: three questionnaires for parents (Q1–Q3), one online focus group with three parents, and one semi-structured interview with the two teachers. The parent focus group was used to reconstruct the home experience with Betta-the-Bee in greater detail, whereas the teacher interview provided contextual information on the classroom work that preceded and followed the home phase.

The number of participating parents varied across the different phases of data collection. Q1 was completed by 32 parents from the six kindergarten sections, Q2 by 19 parents from the same six sections, and Q3 by 12 parents from the two sections directly involved in the Betta-the-Bee pathway. The qualitative follow-up involved three volunteer parents in one online focus group and the two kindergarten teachers in one semi-structured interview.

The research developed through a sequence that linked parents' initial views, schoolwork on the game, and the later home experience. Q1 was administered to document how parents described mathematics in kindergarten, what they expected children to learn, and how they viewed the role of games in mathematical learning. The introduction of Betta-the-Bee then took place in the two selected sections through a gradual school pathway in which children played the game, becoming familiar with the deck, the relevant distinctions represented on the cards, and the rules of the game. Before the Christmas break, parents in the two sections completed Q2, which provided information about games already known and played at home.

During the Christmas holidays, each child took Betta-the-Bee home and was asked to teach it to parents or other adults. This was the central phase of the study, since it made it possible to examine how a mathematical game first experienced at school was later taken up in family interaction. After the holidays, Q3 was administered to the parents of the two sections to capture their views of the game, their child's participation, and the frequency with which the game had been replayed at home. The study was then completed by an online focus group with three parents and a semi-structured interview with the two teachers. These final two sources made it possible to reconstruct the home experience in greater detail and to relate it more closely to the classroom work that had preceded it. The five sources of data served a specific analytic function within the overall design of the study.

Data were analysed thematically across sources. The analysis began with an initial coding of the open responses in Q1 and Q3, the focus group transcript, and the teacher interview. In this first phase, attention was directed to the ways in which parents and teachers described mathematics, mathematical play, the rules of Betta-the-Bee, children's actions, and the home experience with the game. Recurrent expressions and ideas such as counting, logic, explaining the rules, correcting adults, noticing details, and asking questions were therefore retained as initial analytic labels.

A second phase reorganised these initial codes into broader categories aligned with the research questions. This step made it possible to distinguish, on the one hand, parents' initial views of kindergarten mathematics and home play and, on the other, the specific forms of participation and recognition that emerged after the home experience with Betta-the-Bee. The analysis gradually converged on four main claims: parents' initial numerical framing of mathematics, the presence of mathematical play in the home before Betta-the-Bee, children's role in carrying the game home, and the widening of parents' mathematical recognition through the home experience.

The questionnaire data and the data from the focus group and the interview were used in different but complementary ways. Q1 and Q2 were used mainly to identify recurring patterns in how parents described mathematics and play before the home experience. Q3 was used both descriptively and interpretively: descriptively, to make visible compact response patterns concerning parents' recognition of the game and children's participation; interpretively, to identify the themes that required deeper examination in the qualitative material. The focus group and the teacher interview were then examined for episodes in which children were described as explaining the rules, correcting adults, directing attention to relevant variables, or making the structure of the game visible within family interaction.

The findings were constructed through comparison across these sources. Short excerpts were selected as they expressed especially clearly patterns that recurred elsewhere in the dataset, while the tables included in the findings section were used to show the response distributions supporting each claim. The analysis was iterative and involved repeated discussion among the authors to refine category boundaries, test the fit between excerpts and claims, and avoid overly impressionistic interpretations.

5. Findings

The first questionnaire shows that parents do not identify all forms of early mathematics with the same ease. In our data, kindergarten mathematics was recognised above all through visible numerical content: numbers, counting, simple additions and subtractions, quantities, and sequences (see Table 1). Other domains were present, but they appeared less often and with less consistency.

The open responses show this orientation very clearly. Parents wrote "Numbers", "Counting, adding, subtracting", "Numbers from 1 to 10", "Counting at least up to 20", and "Adding

Indicators	**n**
Counting	27
Operations	10
Magnitudes	6
Sets	4
Logical thinking	4
Space and Geometry	3
Time	3
Grouping objects	2
Problem solving	1
Times tables	1

Note. The thematic counts are based on open responses to the question on what children should learn in mathematics at kindergarten.

Table 1. Initial parental framing of mathematics in kindergarten (Q1, n = 32)

and subtracting". A smaller cluster referred to "Preparatory to logical-mathematical thinking", "Logical reasoning", "Sets", "Grouping objects", "Space", or "Geometry". Parents were therefore attributing clear educational value to mathematics in kindergarten. They were also doing so through a language strongly centred on numerical content.

Regarding parents' awareness of school practice, only 11 of the 32 parents reported being aware of mathematical activities proposed at kindergarten, and only 6 mentioned mathematical games used at school. When examples were given, they usually referred to routines in which the numerical dimension was explicit, such as counting the children present in school or using fingers to represent quantity. This pattern is consistent with Muir's [3] observation that parents may be supportive of school mathematics while still having only partial access to the pedagogical logic of classroom practice.

Q1 also suggests that the conditions for a home-school partnership are present, at least in part (Table 2). This is evident from the group of 19 parents who reported playing mathematical games at home with their children while not knowing which mathematical games were being used at school. This result suggests that the main obstacle is not parental unwillingness to engage, but the lack of a shared view of what happens in the classroom. In other words, many parents are already available to support mathematical activity at home, yet they do so without a clear connection to school practice. This makes them a particularly important group for the present study: they are not outside the educational process, but only partially connected to it. From this perspective, Betta-the-Bee can be seen as a promising mediating object, because it offers a concrete way of aligning home activity with school mathematics without asking parents to become teachers themselves.

Combination of parental awareness of mathematical games played at school and habit of playing mathematical games at home	**n**
Aware of the mathematical games proposed at school **and** plays mathematical games at home	6
Aware of the mathematical games proposed at school **but does not** play mathematical games at home	0
Not aware of the mathematical games proposed at school **but** plays mathematical games at home	19
Not aware of the mathematical games proposed at school **and** does not play mathematical games at home	7

Table 2. Parental awareness of mathematical game at school and mathematical game play at home

In Q1, 25 parents reported that they had played mathematical games with their children at home or in other settings (Table 2, rows 1 and 3). Their examples ranged from counting stairs, cars, tiles, fruit, and other everyday objects to small calculations with fingers or household materials, shop and market games, card games, matching activities, and ordering tasks. The home context already included opportunities for counting, comparing, ordering, and reasoning in action. This is very much in line with Cahoon et al. [1], who describe the home mathematics environment as a combination of spontaneous and structured experiences rather than a space of explicit instruction alone.

Q2 sharpens this picture with a closed questionnaire by showing which games were already circulating in the home and which of them parents associated with mathematical development. Table 3 summarises the games that respondents perceived as supporting the development of mathematical skills and/or competences, grouping them into broader game categories.

Game category	**Games included**	**Mentions**
Card-based games	UNO/Solo (9), Rubamazzo/Beggar-my-neighbour (4), traditional playing cards (3), Batawaf (2), Scala 40/Rummy (1), Asino/Old Maid (1)	20
Strategy games	Connect Four (5), Checkers/Draughts (5), Tic-tac-toe (3), Chess (3)	16
Dice- and counting-based games	Dice games (8), Goose Game (6), Bingo/Tombola (2), Monopoly (1)	17
Spatial and puzzle-based games	Labyrinth/Maze games (5), Tetris (3), Tangram (2), Rush Hour (1), Battleship (1)	12
Visual attention and speed games	Dobble (1)	1

Table 3. Games played at home that parents identified as having mathematical content

The combined reading of Q1 and Q2 provides a rich overall picture. In Q1, parents begin to identify some playful activities carried out at home as relevant to children's mathematical learning. In Q2, however, when they are presented with a list of games and asked to select those they perceive as having the strongest mathematical content, their responses shift more clearly towards highly structured games characterised by explicit rules, such as card games, strategy games, and dice games. This pattern suggests that parents do recognise the mathematical potential of such activities, even when they are embedded in playful contexts rather than overtly framed as mathematics. At the same time, it shows that this recognition becomes more explicit when parents are guided by a structured closed-ended questionnaire (Q2), whereas it remains less readily accessible in response to a more open-ended prompt (Q1).

Through this background we can assume that the Betta-the-Bee game entered a setting in which rule-based play was already familiar and often appreciated. The home experience with Betta-the-Bee developed from an asymmetrical starting point. By the time the game reached the family, children had already encountered and played it at school, where they had learned its materials, its rules, and the form of attention it requires. Home play therefore began from prior school experience, and children entered the family setting as participants who already knew how the game worked.

Q3 revisits the themes that emerged from parents' open-ended responses in Q1 (Table 1), in which they described the mathematical content they believed should be learned at kindergarten, by asking them whether they recognised these same mathematical aspects in the game Betta-the-Bee.

The responses reported and grouped in Table 4 show a clear pattern in parents' perceptions of the mathematical potential of Betta-the-Bee. The contents most strongly recognised are logical thinking, followed by counting, space and geometry, time, and grouping. By contrast,

More strongly recognised mathematical contents	**Occurrences (“very useful”)**	**Less strongly recognised mathematical contents**	**Occurrences (“very useful”)**
Logical thinking	10/12	Problem solving	6/12
Counting	9/12	Operations	6/12
Space and geometry	8/12	Measures	5/12
Time	8/12	Sets	4/12
Grouping	8/12	Times tables	3/12

Table 4. Mathematical contents more strongly recognized in Betta-the-Bee

times tables, sets, and measures are less frequently identified, while operations and problem solving occupy an intermediate position. Overall, these results suggest that parents primarily associate the game with early forms of mathematical thinking related to logic, classification, and spatial organisation, rather than with more formal or school-based mathematical contents. In other words, parents began to recognise Mathematics-in-the-Game, alongside Mathematics-in-the-Deck, as a valued form of mathematics enacted in kindergarten.

Game process	**Occurrences (“very much at ease”)**
Explaining the rules	8/12
Remembering where Betta is hidden	10/12
Formulating questions	8/12
Answering questions	9/12
Discarding the cards	9/12

Table 5. Parents’ perceptions of children’s ease with the main game processes in Betta-the-Bee

The responses reported in Table 5 describe children who were already at ease with the structure of the game. Parents generally saw children as comfortable in its practical and verbal dimensions: explaining the rules, managing the phases of play, and keeping the interaction going. The focus group brings what emerged from Table 5 stronger into view: one parent described her daughter as having “asked often” to play Betta-the-Bee, “explaining the rules to different family members”, and added that “the explanation is done by her independently”. Another parent said: “she explained the rules very well”, and after checking the written rules he found that “there was nothing different from what she had explained”. A third parent said that her daughter had explained the rules “in a precise way” and kept proposing the game to “us, friends, or grandparents”. These excerpts show children taking responsibility for starting the activity and shaping it within family interaction. The teacher interview supports this reading from the school side. One teacher described the classroom work on Betta-the-Bee as gradual and underlined how much the children had enjoyed the moment when “we wrote the rules for the parents”. The other reported that the children later “explained very well how to play” even to her. The home competence described by parents grew out of prior classroom work in which the rules, the vocabulary of the game, and its relevant distinctions had already been stabilised.

From the focus group it emerged also that children regulated the game once it had started. One parent said he had been struck by the child’s “precision in telling the rules” and by her “attention to details”, adding that “every now and then I was wrong and I was corrected”. Another recalled her daughter interrupting her with “No, Mum! It isn’t like you said, it’s different”. The teacher interview contains the most compact example of this reversal of expertise: one child reportedly said that “Dad didn’t understand the rules” and that she had had to explain them “many times”. More generally, children explained the rules, restarted the game when needed, corrected adult errors, and kept play aligned with what counted as proper play.

This children's attitude came from the more informed position they have towards the game, since they carried home a rule-governed mathematical practice already established at school. The continuity documented in this study passed through children's capacity to transport that practice into family life and make it workable there.

6. Discussion

The findings allow us to answer the three research questions in a direct way.

Regarding RQ1 – *How did parents initially recognise mathematics and mathematical play in kindergarten and at home?* – the study shows that parents initially recognised mathematics above all through its most visible school-like forms, especially counting and simple operations. Other domains, such as logic, grouping, or spatial reasoning, were present but much less salient in their accounts. At the same time, the data also show that mathematical play was already part of family life before Betta-the-Bee was introduced at home. Parents reported a fairly rich repertoire of games and everyday activities involving counting, comparison, ordering, and strategic action. The point, then, is not that mathematics was absent from the home, but that its recognition was selective. Mathematics was more readily identified when it appeared in explicit numerical form, while other mathematically relevant dimensions were less immediately named. This finding is aligned with the literature [3], highlighting that within the involved parents the poor continuity between school mathematics and home mathematics was not the result of lack of parental interest or lack of playful activity, but a partial overlap between what parents recognised as mathematics and the broader range of mathematical activity valued in kindergarten.

Regarding RQ2 – *How was Betta-the-Bee re-enacted in family interaction, and what role did children play in mediating that re-enactment?* – the findings indicate that the home re-enactment of the game was strongly mediated by children. By the time Betta-the-Bee reached the home, children had already encountered its materials, rules, and relevant distinctions at school. They therefore entered family interaction from a position of prior familiarity and of greater expertise, in some sense reversing the roles in the family interaction, since it is the child that knows more about a specific topic, while usually it is the parent that has the role of expert. Parents' accounts consistently describe children as introducing the game, explaining the rules, proposing it repeatedly, correcting adult misunderstandings, and keeping play aligned with what counted as proper play. The teacher interview supports this interpretation by showing that this competence did not emerge spontaneously at home, but had been prepared through gradual classroom work on the game. The continuity observed here therefore depended on more than the portability of the material. What children carried home was a rule-governed practice already socialised at school. In this sense, children were not simply participating in a home-school connection organised by adults; they were actively making that connection workable within family interaction.

Regarding RQ3 – *How did the home experience with Betta-the-Bee broaden what parents recognised as mathematical in the game?* – the findings suggest a clear shift in the direction of broader mathematical recognition. After the home experience, parents no longer referred mainly to visible numerical content, but more often identified logical thinking, grouping, spatial organisation, and the role of questioning in the game. This does not mean that parents developed a formal or technical discourse about early mathematics. Rather, the threshold of what became noticeable as mathematics appears to have widened. Through play, parents were drawn not only to the mathematical features visible in the cards, but also to the mathematical organisation generated by the rules: asking informative yes/no questions, attending to relevant variables, interpreting answers, and discarding possibilities. The distinction between Mathematics-in-the-Deck and Mathematics-in-the-Game helps clarify this point. The home experience seems to have shifted attention from the former alone to a recognition of the latter as well. This shows that continuity did not concern only the reuse of the same game across settings, but also a broader parental recognition of what counted as mathematical within that activity.

These answers suggest that the educational value of Betta-the-Bee lay not simply in being a game that could travel from school to home, but in being a mathematically structured activity that children could re-enact with others. While large-scale intervention studies have shown the value of adding family mathematics components to classroom-based game interventions [6], the present case study contributes by making visible a possible interactional mechanism through which such continuity may become active: children's mediation of a rule-governed mathematical practice within family life. Continuity was not sustained only through parents' willingness to participate, nor only through the provision of materials, but through children's mediation of a school-shaped mathematical practice within family life. These findings should be treated with a degree of caution, as it must be borne in mind that whilst Betta-the-Bee was indeed carried home by the children, bringing with her a certain perspective on mathematics, the groundwork for this experience had been laid by the teachers and researchers, who had framed the research as a study of children's games with mathematical content. It is therefore possible that this shift in perception, as revealed by the analysis of the children's expertise in play, is underpinned by the awareness raised among parents by teachers and researchers.

7. Conclusions

This study set out to examine three issues: how parents initially recognised mathematics and mathematical play, how Betta-the-Bee was re-enacted at home, and how that experience broadened parental recognition of mathematics. The findings allow a direct answer to each of them.

First, parents initially recognised mathematics mainly through visible numerical content, even though mathematical play was already present in family life through everyday activities and rule-based games. Second, the home re-enactment of Betta-the-Bee was largely mediated by children, who introduced the game, explained and regulated its rules, and sustained it as proper play in family interaction. Third, the home experience broadened what parents recognised as mathematical in the game, shifting attention from visible content alone towards logic, grouping, spatial distinctions, and the strategic role of questioning.

The main contribution of the study lies in the relation between these three findings. What the case makes visible is a specific form of home-school continuity in early mathematics: a mathematically rich activity, first structured at school, can be carried into the home through children's own mediation. In this case, what moved across settings was not only a shared material, but a way of attending, asking, interpreting, and deciding mathematically within play.

The study therefore suggests that children may play a more active role in home-school continuity than is often foregrounded in the literature. Here, they did not simply participate in a connection established by adults but acted as mediators of a rule-governed mathematical practice. This does not reduce the importance of parents or teachers; rather, it shows that continuity may be sustained through a temporary redistribution of expertise within the activity.

These conclusions remain exploratory. The study involved only one kindergarten and a small qualitative post-home dataset. Its value is therefore not statistical generalisation, but the analytic visibility it gives to a specific mechanism. The case suggests that home-school continuity in early mathematics may be strengthened when children are first given enough school experience to appropriate a mathematically structured activity and then are able to re-enact that activity meaningfully in family life. Future research could examine this mechanism in larger samples, in different socio-cultural contexts, and in other beyond-the-classroom environments, including outdoor or community-based settings.

Conflict of interest

The authors declare that there are no conflicts of interest.

References

[1] A. Cahoon, Y. Campver, N. Estévez, C. Jiménez Lira, D. S. Paz García, E. V. Benavides Pando, V. Simms, Parents' views and experiences of the home mathematics environment: A cross-country study, *Infant and Child Development*, **33**(4) (2024), e2497, https://doi.org/10.1002/icd.2497.

[2] S. Phillipson, G. Richards, P. Sullivan, Parental perceptions of access to capitals and early mathematical learning: some early insights from Numeracy@ Home project, in S. Phillipson, A. Gervasoni, P. Sullivan (Eds.), *Engaging families as children's first mathematics educators: International perspectives*, Springer Nature Singapore, (2017), 127–145.

[3] T. Muir, Using mathematics to forge connections between home and school, in V. Kinnear, M. Y. Lai, T. Muir (Eds.), *Forging connections in early mathematics teaching and learning*, Springer Nature Singapore, (2018), 173–190.

[4] J. Streit-Lehmann, Involving parents in games and picture books, in S. Phillipson, A. Gervasoni, P. Sullivan (Eds.), *Engaging Families as Children's First Mathematics Educators: International Perspectives*, Springer Nature Singapore, (2017), 147–163, https://doi.org/10.1007/978-981-10-2553-2_9.

[5] A. Gervasoni, Bringing families and preschool educators together to support young children's learning through noticing, exploring and talking about mathematics, in S. Phillipson, A. Gervasoni, P. Sullivan (Eds.), *Engaging Families as Children's First Mathematics Educators: International Perspectives*, Springer Nature Singapore, (2017), 199–216.

[6] J. M. Young, K. E. Reed, H. Rosenberg, J. F. Kook, Adding family math to the equation: Promoting Head Start preschoolers' mathematics learning at home and school. *Early Childhood Research Quarterly*, **63** (2023), 43–58, https://doi.org/10.1016/j.ecresq.2022.11.002.

[7] V. Kinnear, M. Y. Lai, T. Muir, Forging connections in early mathematics: perspectives and provocations, in V. Kinnear, M. Y. Lai, T. Muir (Eds.), *Forging connections in early mathematics teaching and learning*, Springer Nature Singapore, (2018), 1–15.

[8] R. Casi, C. Sabena, C. Soldano, Fostering logical reasoning with the "Betta-the-Bee" game, in T. Meaney, C. Benz, A. Montone, B. Di Paola, M. G. Fiorentino (Eds.), *Engaging with Mathematics in the Early Years*, (2026), Springer, Cham, https://doi.org/10.1007/978-3-032-16065-2_9.

[9] J. Russo, T. Russo, L. A. Bragg, Five principles of educationally rich mathematical games, *Australian Primary Mathematics Classroom*, **23**(3) (2018), 30–34.

[10] V. Kinnear, E. C. Wittmann, Early mathematics education: a plea for mathematically founded conceptions, in V. Kinnear, M. Y. Lai, T. Muir (Eds.), *Forging connections in early mathematics teaching and learning*, Springer Nature Singapore, (2018), 17–35, https://doi.org/10.1007/978-981-10-7153-9_2.

[11] Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR), *Indicazioni Nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione*, 2012, https://www.mim.gov.it/documents/20182/51310/DM+254_2012.pdf.

[12] R. Casi, C. Sabena, C. Soldano, Dov'è nascosta Betta l'apetta? Giocare con i principi della logica alla scuola dell'infanzia e nei primi anni della scuola primaria. *Didattica della matematica. Dalla ricerca alle pratiche d'aula*, **17** (2025), 114–131, https://doi.org/10.33683/ddm.25.17.5.

[13] T. Núñez, P. Bryant, D. Evans, D. Bell, S. Gardner, A. Gardner, J. Carraher, The contribution of logical reasoning to the learning of mathematics in primary school, *British Journal of Developmental Psychology*, **25**(1) (2007), 147–166, https://doi.org/10.1348/026151006X153127.

[14] R. K. Yin, *Case study research and applications: Design and methods* (6th ed.), SAGE, 2018.