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Mathematical modelling in outdoor learning: exploring analog strategies and cooperative processes in a primary school context

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Abstract: This study explores the design and implementation of an outdoor mathematics learning experience aimed at fostering mathematical modelling, problem solving, and student engagement in a primary school context. The intervention, conducted in a fifth-grade class, was structured around a sequence of outdoor tasks designed by the MathCityMap platform, but adapted to a non-digital format due to school constraints on smartphone use. Grounded in the modelling cycle for the outdoor context, the study examines how alternative instructional strategies can support mathematical learning processes typically mediated by digital tools, as well as the role of cooperative learning in shaping students' participation and understanding. Data were collected through audio recordings, students' written work, and classroom observations, with a qualitative focus on a case study group. The findings show that analog strategies, such as treasure-hunt activities for spatial orientation and structured paper-based hints, can effectively sustain key processes of modelling and problem solving. At the same time, cooperative group work emerged as a crucial factor in promoting engagement and shared reasoning, although it did not always ensure correct interpretation and validation of results. The study highlights the potential of outdoor mathematics education, even in the absence of technology, while also emphasizing the complementary role of digital tools in supporting feedback and assessment processes.

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1. Introduction

The teaching and learning of mathematics are increasingly evolving in response to the challenges of a globalized and technologically mediated society. Traditionally perceived as an abstract discipline, mathematics is now considered in a more dynamic and concrete way, thanks to approaches that emphasize authentic experiences, situated learning, and active student participation. Within this perspective, outdoor learning has emerged as promising pedagogical approach capable of connecting mathematical concepts with meaningful real-world context (Ford, 1986; García-González & Schenetti, 2022).

Outdoor mathematics education goes beyond simply relocating lessons outside the classroom. Rather, it represents a pedagogical vision centered on direct experience, interaction with the environment, and embodied learning processes (Kelton & Ma, 2018). Through activities situated in natural or urban settings, students are encouraged to explore mathematical ideas through movement, observation, measurement, and collaboration, fostering a more concrete and experiential understanding of mathematics (Taranto et al., 2021). In these contexts, mathematics becomes a tool for interpreting and describing reality, supporting the development of problem-solving abilities, critical thinking, and collaborative/cooperative skills.

In recent years, growing attention has been devoted to outdoor mathematics activities such as math trails, in which students solve contextualized mathematical problems connected to elements of the surrounding environment. Digital tools have further expanded the possibilities for designing and implementing these activities, supporting exploration, immediate feedback, and situated problem solving (Buchholtz et al., 2020; Fessakis et al., 2018; Ludwig & Jesberg, 2015). At the same time, research has increasingly highlighted the educational potential of mathematical modelling processes developed within authentic contexts. However, the literature also presents some limitations. First, many studies focus primarily on technology-supported implementations, while less attention has been devoted to contexts in which digital tools are unavailable or restricted. Second, although outdoor mathematics is frequently associated with increased engagement and meaningful learning, findings are not always unequivocally positive, as the effectiveness of outdoor activities strongly depends on task design, teacher guidance, and the balance between exploration and mathematical focus. Finally, limited research has investigated how cooperative learning dynamics and non-digital scaffolding strategies support students' mathematical modelling processes during outdoor activities in primary education.

To address these gaps, the present study investigates the implementation of non-digital math trails in a fifth-grade primary school context. In particular, the study examines how alternative teaching strategies can replace digital tools while maintaining the educational value of outdoor mathematics activities, and how cooperative learning influences students' participation and mathematical understanding during the modelling process.

2. Literature review

2.1. Outdoor mathematics education

Outdoor learning has increasingly attracted attention within mathematics education research as a way of promoting meaningful, experiential, and embodied learning processes. By engaging directly with natural and urban environments, students can explore mathematical concepts through observation, movement, and interaction with real-world contexts (Zender et al., 2020). Mathematics, experienced in this way, becomes a dynamic activity connected to authentic and everyday contexts, fostering a deeper understanding of mathematical concepts and supporting the construction of more solid knowledge (Jablonski, 2025).

Several studies highlight how outdoor mathematics activities may positively influence students' engagement, motivation, and participation. Activities such as measuring objects in the environment, estimating distances, identifying geometric configurations, or collecting and representing data allow students to connect abstract concepts with concrete experiences (Barbosa, 2006; Schukajlow et al., 2022; Wiehe et al., 2025). Outdoor contexts may also encourage collaborative problem solving and peer interaction, contributing to the development of communication and cooperation skills (Fägerstam & Blom, 2013; Johnson & Johnson, 1999). From a pedagogical perspective, outdoor mathematics is often associated with constructivist and inquiry-based approaches in which students actively participate in the construction of knowledge. In these settings, the teacher assumes the role of facilitator, supporting exploration, discussion, and reflection rather than simply transmitting information (Hmelo-Silver et al., 2007). Furthermore, embodied and multisensory experiences may support students' concentration, emotional involvement, and mathematical understanding by integrating cognitive, physical, and social dimensions of learning (Ferrari & Taranto, 2024).

Outdoor mathematics education also promotes interdisciplinary connections between mathematics and other subjects such as science, geography, and art, helping students perceive mathematics as meaningful and relevant within broader real-world contexts (Beames et al., 2012).

At the same time, the literature also reports some challenges and mixed findings concerning outdoor mathematics education. Some studies suggest that outdoor settings alone do not automatically increase students' interest, enjoyment, or mathematical understanding. Rather, the effectiveness of outdoor activities appears to depend strongly on factors such as task authenticity, the quality of scaffolding, and the organization of collaborative work (Hartmann & Schukajlow, 2021). Similarly, recent research comparing different modelling settings highlights that outdoor environments may foster specific modelling processes, while also introducing additional challenges related to data collection, interpretation, and cognitive management (Jablonski, 2025).

2.2. Math trails: digital and non-digital approaches

Among the different approaches developed within outdoor mathematics education, outdoor mathematics trails, or math trails, represent one of the most widespread and established formats. Math trails are introduced as structured itineraries in natural or urban environments where students engage with authentic problem situations closely connected to the context. These trails transform real space into a learning environment in which mathematics emerges through observation, measurement, and interpretation of elements in the surrounding environment, fostering processes of modeling, argumentation, and problem solving (Jablonski, 2025).

Traditionally, math trails have been implemented through paper-based tasks and teacher-guided activities (Blane & Clarke, 1984). More recently, digital technologies have introduced new possibilities for designing and managing outdoor mathematical experiences. One of the most prominent examples is MathCityMap (https://mathcitymap.eu/en/), widely used for designing and implementing the math trails. It is a geolocation-based digital platform that guides students along a trail composed of situated mathematical "tasks": using mobile devices, students reach specific points in physical space and, once there, activate the problem to be solved. Geolocation thus directly links mathematical content to the real context, making the experience highly situated and interactive. Another distinctive feature is the presence of progressive hints, which support students when they encounter difficulties without revealing the solution, thereby fostering scaffolding processes and autonomy. Furthermore, the system includes mechanisms for immediate validation of answers: students receive automatic feedback, often based on numerical tolerances, which allows them to verify the correctness of their results and reflect on the procedures used (Jablonski, 2024).

From a pedagogical perspective, math trails generally develop according to a cooperative methodology. Students work in small groups, within which specific roles are assigned. For example, one student manages the device and navigation, another is responsible for measurements, and another coordinates calculations and discussion, thus fostering active participation and shared responsibility among all members. This organization promotes not only the learning of mathematical content but also the development of transversal skills such as cooperation, communication, and critical thinking. In this way, math trails, supported by digital tools such as MathCityMap, create a dynamic learning environment in which experience, technology, and social interaction are integrated, making mathematics more accessible, meaningful, and deeply connected to reality.

Several studies report positive effects of digital math trails on students' engagement and motivation (Ludwig et al., 2021; Jablonski, 2024). However, scholars also underline that the educational effectiveness of these tools depends not only on technology itself but also on the pedagogical quality of task design and classroom orchestration (Ratnayake et al., 2020). Furthermore, recent restrictions on the use of mobile devices in some school contexts raise the need to investigate alternative ways of implementing math trails without losing their exploratory, cooperative, and contextualized nature. From this perspective, non-digital adaptations may represent a relevant area for further research.

Within this framework, the present study explores the implementation of a non-digital outdoor math trail in a primary school context. While inspired by the MathCityMap approach to task design and outdoor mathematical exploration, the activities were redesigned in an analog format due to restrictions on the use of mobile devices from the students. In particular, the study investigates how key features of math trails, such as contextualized problem solving, cooperation, and scaffolding through discussion and comparison, can be maintained in the absence of digital technologies. In doing so, the study aims to contribute to current discussions on the adaptation of outdoor mathematics activities to different educational contexts and practical constraints.

3. Theoretical framework and research questions

Mathematical modeling represents a central element in mathematics education, as it makes it possible to connect abstract concepts with real-world situations, fostering meaningful and contextualized learning (Blum et al., 2007; Kaiser & Sriraman, 2006). Through modeling, students are required to interpret authentic problems, select relevant information, and translate situations into mathematical terms, thereby developing problem-solving skills and critical thinking.

Unlike traditional exercises, modeling problems are characterized by their open and unstructured nature, requiring students to actively construct solution strategies. This process involves moving from a real situation to a mathematical representation through the identification of variables, relationships, and constraints, which can be expressed through formulas, graphs, or diagrams (Kaiser, 2007). A distinctive feature of modeling is its iterative nature: students formulate hypotheses, build models, and compare them with reality, revising and adapting them progressively. This dynamic fosters not only conceptual understanding but also cognitive flexibility and the ability to reflect on one's own thinking processes.

Over the past two decades, research on mathematical modelling has considerably expanded, moving beyond predominantly cognitive interpretations of modelling processes toward sociocultural, embodied, and situated perspectives (Borromeo Ferri, 2018; Greefrath et al., 2023). Recent studies increasingly highlight the importance of authentic contexts, collaboration, multimodality, and inquiry-based environments in fostering students' modelling competencies (Maass et al., 2019; González-Martín et al., 2021).

Within the classical modelling tradition, the modelling cycle proposed by Blum and Leiß (2007) represents one of the most influential theoretical frameworks. The model conceptualizes modelling as a cyclical process involving several interconnected phases: understanding and simplifying a real-world situation, mathematization, mathematical work, interpretation, and validation of results. The cyclical nature of the process emphasizes that modelling is not linear, but rather characterized by continuous transitions between reality and mathematics through revisions, reinterpretations, and adaptations.

More recent research has further emphasized that modelling processes are deeply shaped by the environments in which they take place. In outdoor mathematics activities, students do not simply apply mathematical concepts to external situations; rather, they interact bodily, socially, and materially with the surrounding environment while constructing mathematical meaning (Ferrari & Taranto, 2024). Outdoor contexts therefore transform modelling into an embodied and situated activity in which movement, perception, spatial orientation, and interaction with real objects become integral parts of the learning process. From this perspective, modelling is no longer interpreted solely as a transfer between a reality domain and a mathematical domain, but also as a process mediated by bodily engagement, collaborative interactions, and the ways students orient themselves within the environment (Ferrari & Taranto, 2024).

For this reason, the present study adopts the adaptation of the modelling cycle for outdoor contexts (Figure 1) proposed by Jablonski and Bakos (2022), which explicitly emphasizes the transitions between reality and mathematics during outdoor mathematical activities. In this model, students continuously move between understanding and structuring the outdoor context, mathematizing real situations, interpreting mathematical results, and validating solutions within the physical environment itself. The framework is particularly suitable for analysing math trails because it highlights how modelling processes emerge through direct interaction with space, measurements, estimation, and contextual constraints.

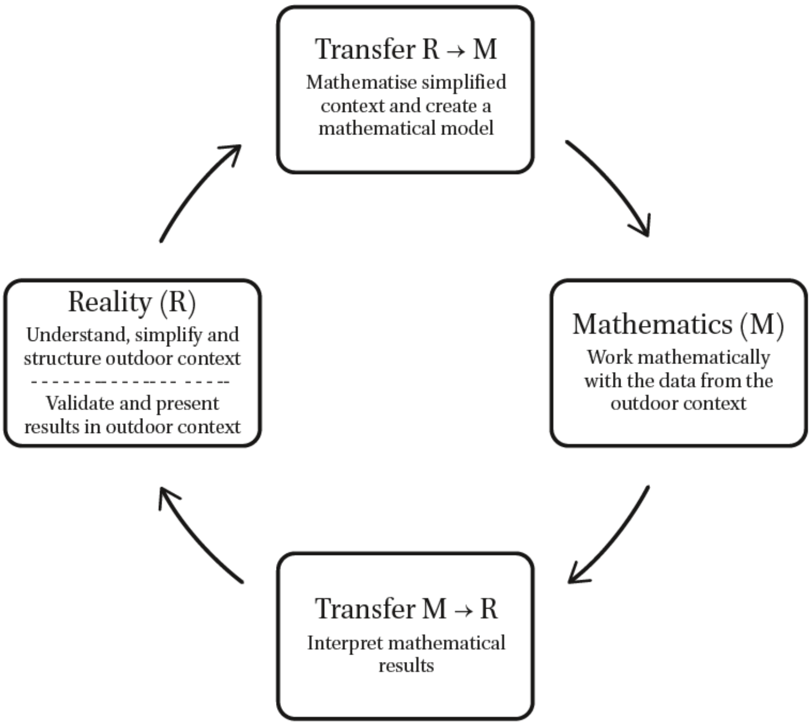

Figure 1. Modelling cycle for the outdoor context (from Jablonski & Bakos, 2022, p. 4)

At the same time, modeling activities in outdoor contexts often take place through cooperation among students, fostering cooperative learning dynamics. Peer interaction allows students to articulate their reasoning, negotiate meanings, and construct shared solutions. In this sense, the social dimension of the modeling process intertwines with the cognitive one, contributing to the development of transversal skills such as communication, cooperation, and argumentation (Jablonski, 2025). Furthermore, the introduction of problem-solving support systems, such as

structured hints, plays a significant role. These tools can be interpreted as forms of scaffolding that support students through the different phases of the modeling cycle, particularly in moments of difficulty, without replacing the process of constructing the solution (Taranto & Jablonski, in press). In the absence of digital tools, the adoption of alternative strategies still allows this support function to remain active, promoting students' autonomy and reflection.
In the present study, mathematical modelling is therefore interpreted as a situated, cooperativa, and embodied process. The integration of outdoor learning, cooperative interactions, and non-digital scaffolding strategies provides a particularly meaningful context for investigating how students construct mathematical understanding while engaging with authentic problems in real environments.
In light of these theoretical considerations, the present study aims to investigate the following research questions:
i) Which teaching strategies can effectively replace digital tools without reducing the quality of mathematical learning in math trails?
ii) How does cooperative learning influence students' participation and mathematical understanding during math trails activities?

4. Research context and method

The design of the instructional intervention is part of a math trail, with the aim of integrating concrete experience and mathematical reflection, promoting active, cooperative, and contextualized learning. The classroom context is understood not only as a physical space, but as a dynamic learning environment, constructed through relationships, interactions, and shared experiences, which also extends to the school's outdoor spaces. From this perspective, the activities were designed to enhance the connection between field exploration and classroom formalization, fostering the development of both disciplinary and transversal skills.
The study was conducted in a fifth-grade class at the "Via Puglia" primary school of the "Leonardo Sciascia" Comprehensive Institute in Misterbianco (Catania), located in a peripheral area of the municipality. The school involved offers numerous spaces and resources, including a gym, an outdoor sports field, a library, laboratories, and environments dedicated to innovative activities: elements that made it possible to implement a structured math trail. The second author of the article carried out a teaching internship in this class during the fifth year of the Primary Education degree program; the contribution presented here derives from the results of her master's thesis, conducted under the supervision of the first author.
The participating class, a fifth-grade group of 23 students, ranges in ages from 10 to 11; 10 of the students are girls and 13 are boys. The class shows an overall average level of mathematical proficiency and is characterized by a certain heterogeneity, which allowed for observing the effectiveness of the proposed activities across different levels of competence. One student with Special Educational Needs is present, supported by a special education teacher, which required particular attention in designing inclusive and accessible activities. The students had already acquired arithmetic, geometric, and logical reasoning skills, which were further consolidated through the proposed math trail. The intervention took place during regular mathematics lessons over three days, between late April and early May 2024.
The activities are structured as a math trail composed of authentic tasks designed through the MathCityMap platform; however, due to the school regulation prohibiting the use of smartphones during instructional activities, it was not possible to use the application in its digital form. It was therefore necessary to adopt alternative approaches to compensate for the absence of features such as geolocation and hints, while maintaining the structure and educational objectives of the trail. To this end, a printed booklet containing the entire math trail was downloaded from the MathCityMap platform (available at the following link: https://url-shortener.me/KJG4, including a map, task descriptions, images of reference objects, instruc-

tions, and spaces for carrying out calculations and recording solutions, with the aim of connecting disciplinary content to real-life situations.
The tasks cover the main thematic areas outlined in the Italian national curriculum (Numbers; Space and figures; Relationships, data and predictions) and address concepts such as area, perimeter, angles, length, volume, fractions, and time measurement. Presented as authentic tasks, they require students to observe, measure, compare, and calculate using concrete tools (i.e., measuring tape, protractor, . . .), fostering modeling and problem-solving processes in real contexts. The entire math trail is oriented toward the development of transversal skills such as logical reasoning, critical observation, communication, and cooperation.

4.1. Design of the instructional intervention

The design is structured into three main phases.

4.1.1. Design and implementation of Phase 1

Attention was given to the initial activities, designed to prepare students both from an organizational and a relational perspective. Given the prohibition on smartphone use and the resulting impossibility of accessing MathCityMap's geolocation features, it was necessary to design alternative activities aimed at fostering spatial orientation. To this end, the introductory phase was structured into two distinct sub-phases: an initial treasure hunt activity aimed at developing orientation skills and familiarization with the environment, followed by a cooperative physical game, "Untangle the Knot", intended to strengthen group relational dynamics.
In the treasure hunt, through riddles and multidisciplinary challenges distributed across the school spaces, students were guided to orient themselves within the school environment and to become familiar with the locations where the mathematical tasks would later take place. The stops of the treasure hunt did not coincide exactly with the locations of the MathCityMap tasks, but were situated in nearby areas, so as to encourage gradual familiarization with the spaces involved in the math trail. Organized into two teams, the children followed different routes, solving puzzles and completing challenges to progress in the game. This activity fostered not only spatial orientation but also active engagement and early collaborative dynamics among peers. Furthermore, this phase allowed the second author (i.e., the experimenter) to observe the interactions among the students and to form heterogeneous group suitable for the subsequent phase involving the mathematics trail.
The "Untangle the Knot" game aimed instead to introduce students to cooperative work. In this activity, children, divided into groups, were involved in a physical problem-solving task: untangling a human knot without letting go of each other's hands. Within each group, 5 children stood in a circle holding hands and served as the "inner" members, while 1 or 2 classmates remained outside the circle as observers and facilitators. To create the knot, the children inside the circle turned their arms and hands in a "whirling" motion (Figure 2). At the experimenter's signal, the hands had to be extended towards the center of the circle along an imaginary vertical line (Figure 3). Starting from the top, the hands were to be held in pairs until a human tangle was formed. The participants outside the circle were responsible for carefully observing the configuration of the tangles and providing strategic instructions to their classmates inside the circle, while the latter carried out the necessary actions to untangle the knots in their hands. The instructions mainly involved spatial and topological movements, such as "raise your right arm", "turn to the left" or "pass under your arm". This experience made it possible to bring out group dynamics, roles, and communication patterns, preparing students for the cooperative work required in the following activities and fostering the development of shared strategies.

Figure 2. "Whirling" motion

Figure 3. Arms extended toward the center of the circle

4.1.2. Design and implementation of Phase 2

The second phase represents the core of the pathway and involves the implementation of a math trail structured into 12 tasks. The tasks can be accessed through the printed booklet https://url-shortener.me/KJG4 or via the free MathCityMap app by entering the code "3920683".

Students, divided into groups of 3-4 members, take on specific roles: the secretary, responsible for completing the printed booklet; the measurer, in charge of using measurement tools (rigid ruler, measuring tape, protractor, calculator); the navigator, who guides the group using the map; and, in groups of 4 members, the timekeeper, whose task is to monitor the duration of the activities and signal when it is time to move on to the next task after approximately 10–15 minutes of work.

To address the impossibility of using the MathCityMap application and accessing digital hints, an alternative system based on envelopes containing paper slips was introduced. In case of difficulty, each group could request a limited number of hints by turning to a designated reference figure: the experimenter, acting as the researcher, or one of the classroom teachers (the mathematics teacher and the support teacher), who primarily acted as observers and, in this phase, as facilitators of the activity. The hints, provided one envelope at a time for a maximum of three per task, corresponded to those originally designed within the tasks on MathCityMap. This system served as a mediating tool in the problem-solving process, supporting students' autonomy without oversimplifying the task and fostering the development of solution strategies.

The structure of the math trail, with staggered starting points for each group, ensured effective space management and avoided overlap, allowing each group to work independently. Although the MathCityMap platform offers useful tools for managing flows, even in the absence of technology the experimenter organized the start of the activities by assigning each group a different initial task and planning a cyclical progression along the trail. This solution made it possible to optimize movement and prevent multiple groups from concentrating on the same task at the same time, maintaining a good balance between operational autonomy and overall activity management.

4.1.3. Design and implementation of Phase 3

The third phase takes place in the classroom and is dedicated to reflection and the re-elaboration of the experience. Through a whole-class discussion, the completed tasks are revisited, with particular attention to those that posed greater difficulties. The experimenter guides the discussion through prompting questions, encouraging students to articulate the strategies they adopted, compare solutions, and reflect on errors. This moment makes it possible to consolidate learning and to make explicit the mathematical processes activated during the experience. Finally, an evaluation of the experience is proposed, in which students are invited to express their perspectives on both the emotional and cognitive aspects of the trail. This phase values the subjective dimension of learning and provides useful insights for assessing the effectiveness of the intervention.

4.2. Data analysis

In analyzing the data related to Phase 2, a qualitative case study approach was adopted, combining thematic analysis with theory-driven coding informed by the modelling cycle for outdoor contexts (Jablonski & Bakos, 2022). One group was selected as a case study in order to examine in depth the interaction dynamics and problem-solving processes enacted during the math trail. In particular, Group A (one of the five groups involved), composed of three students with heterogeneous levels of competence, was selected because its balanced composition was considered meaningful for observing processes of negotiation, cooperation, and shared knowledge construction in an authentic context.
Within the data corpus, two tasks were identified as particularly significant in relation to the research questions (Table 1): Task 2 (Cultural exchange), in which difficulties emerged, and Task 6 (The time of the swings), which was completed successfully.
Both tasks, like all tasks that constitute a math trail, required not only the use of the data provided in the instructions, but also the direct collection of certain information through observation and measurement of the surrounding environment. To make this point clearer and assist the reader, the table below indicates in square brackets which data must be measured on-site and which must instead be gathered through direct observation, as they are intentionally not provided in the task instructions, in line with the design principles of outdoor math trail tasks.

The analysis aimed at examining students' situated mathematical activity within the outdoor learning environment. In line with the theoretical framework presented above, particular attention was devoted not only to the modelling processes activated during task resolution, but also to the cooperative, contextual, and embodied dimensions of students' interactions. The selection of the episodes was guided by the presence of rich interactions, explicit negotiation of modelling strategies, and the use of the hint system, which represented a central element of the instructional design. In particular, Task 6 makes it possible to observe how cooperation and scaffolding through hints supported the successful completion of the modeling process, whereas Task 2 allows for the analysis of a situation in which, despite the presence of cooperative strategies and support, the modeling cycle remains incomplete, especially in the phases of interpretation and validation.
The data corpus consisted of video recordings, field notes collected during the activities, and students' written productions contained in the printed booklets. The analysis focused on episodes characterized by negotiation of strategies, use of hints, cooperative interactions, and moments of difficulty or revision during problem solving. An iterative thematic analysis was conducted through repeated observation of the selected episodes. The coding process was informed by the modelling cycle for outdoor contexts (Jablonski & Bakos, 2022), with particular attention to transitions between understanding the real situation, mathematization, interpretation, and validation. The coding and interpretation of the episodes were discussed

Task	Instruction	Sample Solution	Hints
Cultural Exchange 	A school welcomes a group of 10 foreign children for a cultural exchange. Consider the sofa and the pouf as a single seating space, and note that there are two sofas and two poufs. Knowing that each seated child occupies 40 cm of length, how many children remain standing?	Sofa length (obtained through direct measurement) = 90 cm. Pouf length (obtained through direct measurement) = 45 cm. Total seating space: $$90 + 45 = 135 \text{ cm}$$ Number of children who can sit: $$135 \div 40 = 3$$ Since there are two sofas and two poufs: $$3 \times 2 = 6$$ Therefore, 6 children can sit and $$10 - 6 = 4$$ children remain standing.	1. Measure the length of the sofa and the pouf. 2. To determine how many children can sit, divide the available seating space by the space occupied by one seated child.
The Time of the Swings 	A group of 24 children goes to the playground to use the swings. At 4:40 p.m., all swings are occupied and each child uses a swing for 10 minutes. At exactly 4:50 p.m., they take turns with other children. How many more children will be able to use the swings until 5:30 p.m.?	There are 4 swings in the playground (obtained through direct observation). From 4:40 p.m. to 5:30 p.m., 50 minutes pass. $$50 \div 10 = 5$$ groups of children can take turns. Therefore, $$5 \times 4 = 20$$ children can use the swings.	1. Count the number of swings in the playground. 2. Pay attention to the elapsed time.

Table 1. Examples of mathematical tasks proposed in the playground.

repeatedly by the authors in order to ensure coherence between the analytical categories and the theoretical framework.

5. Implementation of the study

The instructional intervention took place between late April and early May 2024, for a total of 6 hours divided into three sessions over the course of one week. As mentioned above, two sessions were conducted outdoors, promoting experiential learning, while the final session took place in the classroom, with the aim of revisiting and consolidating the knowledge acquired.

5.1. Phase 1

The first day was designed to familiarize students with the school environment and to prepare them for cooperative work required in the subsequent mathe trail activities. Since digital devices could not be used and students therefore had no access to MathCityMap geolocation tools, a treasure hunt was organized to foster spatial orientation skills.
Students were divided into two teams and followed different routes through the school spaces by solving riddles and completing multidisciplinary tasks. The activity required students to interpret clues, identify landmarks within the environment, and move through the school using spatial references. In this way, students progressively developed orientation skills while collaboratively negotiating hypotheses and solutions. The riddles guided the groups toward locations such as doors, stairs, windows, and gates, encouraging direct interaction with the surrounding environment (e.g., Figure 4).

Figure 4. One of the teams searching for the window

The second part of the first day focused more explicitly on cooperation and spatial communication through the "Untangle the Knot!" game. Students worked in groups, with some children forming a human knot and others acting as external guides. The task required the group to untangle the structure without releasing their hands.
During the activity, students continuously used topological and directional language such as "go under," "turn to the left," "raise your arms," and "step over," demonstrating the active use of spatial references to coordinate collective actions (see Figures 5 and 6). The effectiveness of the task depended on the group's ability to communicate clearly, negotiate strategies, and cooperate in solving the problem.
The activity therefore represented an authentic cooperative problem-solving situation in which spatial orientation, verbal communication, and mutual coordination were strictly interconnected. Consistent with cooperative learning principles (Johnson & Johnson, 1999), students' success depended on the group's capacity to act collectively rather than on individual performance alone.

Overall, the first phase allowed students to become familiar with the school environment, strengthen spatial orientation skills, and develop cooperative strategies that were fundamental for the subsequent math trail activities.

Figure 5. Movements of the first group during the “Untangle the Knot” game

Figure 6. Movements of the second group during the “Untangle the Knot” game

5.2. Phase 2

The second day of the intervention was dedicated to carrying out a math trail designed using MathCityMap. The session began in the classroom with an introductory explanation, during which the experimenter illustrated the operational procedures of the activity, with the aim of actively and consciously engaging the students. The intention was to promote dynamic learning, capable of combining theoretical knowledge with concrete applications in real contexts.

Subsequently, the students were divided into five groups (A, B, C, D, E), each consisting of 3 or 4 members. Within each group, specific roles were assigned: secretary, measurer, navigator, and, in groups of four, the timekeeper. This organization encouraged active and responsible participation from all students, structuring the work in a cooperative way.

As digital devices could not be used, the groups oriented themselves using a printed map of the tasks (Figure 7), following the math trail with its 12 activities distributed across the school spaces. In case of difficulty, each group could request hints from a previously assigned reference figure (i.e., the experimenter or one of the teachers). In case of difficulty, each group could request hints from a previously assigned reference figure (i.e., the experimenter or one of the teachers).

The following analysis focuses on the implementation of Tasks 2 and 6 by Group A, composed

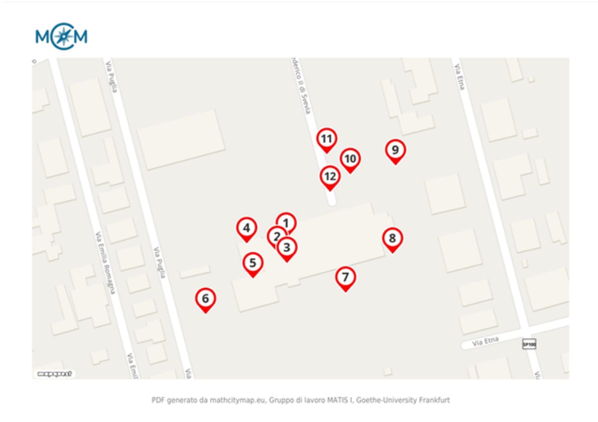

Figure 7. MathCityMap map with the tasks to be completed

of three students with well-defined roles: Giuly, the secretary; Alex, the measurer; Viky, the navigator.

5.2.1. Task 2 – Cultural exchange

During the implementation of the task "Cultural exchange", Group A immediately showed intense peer interaction, characterized by attempts at shared interpretation and negotiation of strategies. After reading the instructions, the students correctly identified the need to take measurements, activating the measurer:

G.: "You have to measure the sofa"

V.: "You have to measure everything, so the pouf as well, I think"

Once the data had been collected, the group began discussing the operations to be carried out, but showed uncertainty in structuring the problem mathematically. Proposals followed one another rapidly, without a clearly shared direction:

A.: "The total length is 136 cm... so now we have to do 136 ÷ 40"

V.: "But we have to consider that there are two"

G.: "Then we have to do 136 + 136"

V.: "So 136 × 2"

This phase highlights how the students began to construct a model of the situation but were unable to stabilize a coherent representation of the problem. Peer interaction remained active but did not immediately lead to an effective solution strategy:

A.: "Should we do 400 ÷ 272?"

G.: "Maybe the other way around... 272 ÷ 400"

A turning point occurred with the use of hints. After recognizing the limited usefulness of the first hint (see Table 1), the group accessed the second one (see Table 1), which oriented their reasoning toward a correct relationship between the quantities:

V.: "It was complicated... now I understand what we have to do"

Despite this, a significant issue emerged in the final phase. The students correctly performed the calculations but accepted a result that was not consistent with the real context:

V.: "3.2 remain standing"

This episode is particularly relevant from a theoretical perspective. The group was able to activate several phases of the modeling process, from initial understanding to mathematization and mathematical work, but showed difficulties in the interpretation and validation phase. The acceptance of a non-integer value when referring to a number of people reveals a lack of reflection on the plausibility of the solution, interrupting the modeling cycle.

At the same time, the task highlights the ambivalent role of cooperation: while peer interaction supports the construction of reasoning and helps overcome moments of impasse, it does not automatically ensure metacognitive control over the final result. In this sense, the hint system appears to be an effective support for the mathematization phase, but insufficient to sustain validation processes, which require a more advanced level of mathematical awareness.

5.2.2. Task 6 – The time of the swings

In the task "The time of the swings", Group A engages with a problem situation that requires the integration of spatial and temporal information. In an initial phase, the students correctly identify some elements of the context, such as the number of swings available, but struggle to coordinate the data to construct a solution strategy:

G.: "There are 4 swings"

V.: "Should we do 24 ÷ 4?"

A.: "But what is the question?"

This uncertainty leads the group to request a hint. The first hint (see Table 1), related to counting the swings, does not provide new information, while the second (see Table 1) introduces a decisive element:

V.: "So 4 children every 10 minutes can use the swings"

From this point, a significant change in group dynamics can be observed. The students begin to progressively construct the solution, making each step explicit and coordinating their reasoning:

G.: "In 10 minutes 4 children use the swings [. . .] if another 10 minutes pass, that's 8 children"

A.: "And it's 5:10 p.m."

G.: "At 5:20 p.m. we reach 16 children"

G., V., A.: "At 5:30 p.m. it's 20 children!"

The process develops through continuous interaction among group members, in which each student contributes to building the solution, integrating and completing each other's reasoning. In this case, cooperation goes beyond simply sharing ideas, becoming a genuine process of co-construction of knowledge.

From a theoretical perspective, the group is able to complete all phases of the modeling cycle. After an initial difficulty in understanding and structuring the problem, the hint helps guide the mathematization correctly, transforming the real situation into an organized temporal sequence. The mathematical work develops through an iterative progression, while the interpretation of the result appears consistent with the context.

In this case, the hint system performs a particularly effective scaffolding function, supporting the group in overcoming a cognitive obstacle without directly providing the solution. Peer cooperation also contributes to distributing the cognitive load and making the reasoning steps explicit, fostering a shared understanding of the problem.

The comparison with the previous task highlights how, despite having the same tools and instructional organization, the outcome of the process depends on the group's ability to integrate the different phases of modeling, particularly those related to interpretation and validation, which are crucial for successful problem solving.

5.3. Phase 3

The intervention continued the following day with the third session, held in the classroom and dedicated to reflection and discussion. In this concluding phase, the experimenter guided the students in analyzing the completed tasks, following a chronological order and focusing in particular on those that had presented greater difficulties. This work was made possible through audio recordings of Group A's conversations during the activities and the review of the booklets completed by all groups. Based on these materials, targeted questions were prepared, also differentiated according to the roles assumed by the students during the math trails.

Within this reflective phase, it was possible to interpret the processes enacted by the students through the modeling cycle for outdoor contexts, which proved useful for analyzing the degree of completeness of the problem-solving process.
The analysis also highlighted how, in the absence of the immediate validation typical of the digital MathCityMap environment, errors tend to emerge mainly during the subsequent review phase. This made it possible to reflect on the value of the adopted hint system (namely, the use of envelopes) which proved effective in supporting the problem-solvig process, although it did not fully replace the feedback functionalities offered by the digital platform.
These findings also resonate with research highlighting the complexity of modelling processes for learners, particularly in authentic and outdoor contexts. Although the present study adopts the outdoor adaptation of the modelling cycle proposed by Jablonski and Bakos (2022), this framework still reflects the challenges originally discussed within the modelling tradition developed by Blum and Leiß (2007), especially regarding students' difficulties in coordinating interpretation, validation, and transitions between the outdoor context and mathematical representations. From this perspective, the hint system adopted in the present study can be interpreted as a form of scaffolding that supported students in managing the complexity of the modelling activity while preserving their autonomy during problem solving.
The final phase was enriched by an open discussion, during which students expressed their perceptions of the experience. A high level of engagement and appreciation for the activities emerged, particularly for those involving logic, movement, and cooperation. The session concluded with the awarding of participation certificates and a collective debriefing moment, which acknowledged and valued the students' effort and active participation.

6. Discussion, implications and conclusions

The findings of the present study are consistent with recent research on outdoor mathematics education and situated mathematical modelling, which emphasizes the importance of authentic contexts, collaboration, and embodied interaction in supporting students' construction of mathematical meaning (Ferrari & Taranto, 2024; Jablonski, 2025; Kelton & Ma, 2018). At the same time, the study extends previous research by exploring how these processes can also be supported in the absence of digital technologies through alternative instructional and scaffolding strategies.
Considering the research questions, the results allow for several relevant considerations.
Regarding the first question, "Which teaching strategies can effectively replace digital tools without reducing the quality of mathematical learning in math trails?", the data show math trails can be effectively implemented even in the absence of the direct use of technology. Some instructional choices proved effective in compensating for the main features of the MathCityMap application. The initial treasure hunt played a key role in replacing digital geolocation, enabling students to become familiar with the space and develop orientation skills. Similarly, the system of envelopes containing hints represented a valid alternative to digital hints, supporting the problem-solving process through forms of gradual scaffolding.
These findings are in line with previous studies highlighting the importance of orientation, contextualization, and task design in outdoor mathematics activities (Fessakis et al., 2018; Ratnayake et al., 2020). However, while much of the existing literature focuses on technology-supported implementations, the present study suggests that some key educational affordances of math trails (such as exploration, situated problem solving, and scaffolding) can also be preserved through non-digital adaptations. More broadly, the findings support situated and embodied perspectives on mathematical modelling, according to which interaction with physical space and direct engagement with the environment play an important role in the construction of mathematical meaning (Ferrari & Taranto, 2024; Kelton & Ma, 2018).
A particularly relevant point of difference concerns the management of validation and feed-

back processes. The whole-class discussion phase, in fact, represents an essential moment in math trails, as it allows students to reflect on the processes activated, compare the strategies adopted, and consolidate learning. However, in the absence of immediate feedback, typical of the digital environment, the experimenter was not able to obtain an immediate overview of the progress of the activities and the groups' results. This led to a greater workload during the analysis phase, requiring a detailed review of the booklets to reconstruct the processes and identify the difficulties encountered by the students.

Regarding the second question, "How does cooperative learning influence students' participation and mathematical understanding during math trails activities?", the results highlight the central role of the cooperative dimension. Working in small groups, structured through the assignment of roles, fostered the active participation of all students and the shared construction of solutions. In particular, the initial "Untangle the Knot" game helped make the meaning of cooperation explicit, supporting students in understanding the importance of their role within the group and the need to coordinate with others to achieve a common goal.

The analysis of the tasks shows how cooperation supports the modeling process, allowing students to articulate their reasoning, engage in discussion, and negotiate strategies. However, it also emerges that cooperation alone does not automatically ensure successful completion of the modelling cycle: as shown in Task 2, the group, despite working cooperatively, was not able to correctly validate the result. This suggests that cooperation must be accompanied by careful instructional guidance, capable of supporting even the more complex phases of the modeling cycle, particularly those related to interpretation and validation.

This result confirms previous research on cooperative learning and outdoor mathematics education, according to which peer interaction may support reasoning processes, negotiation of meanings, and shared construction of mathematical understanding (Johnson & Johnson, 1999; Fägerstam & Blom, 2013). At the same time, the findings also extend existing research by showing that cooperation alone does not automatically ensure successful completion of the modelling cycle, particularly in the phases of interpretation and validation.

A further relevant aspect concerns assessment, understood not only as the evaluation of the final product, but as the analysis of the process. Observing interactions, adopted strategies, and encountered difficulties made it possible to gain a deeper understanding of learning processes, in line with the model proposed by Jablonski and Bakos (2022). In this sense, assessment takes on a formative function, guiding both instructional action and students' reflection.

Despite the positive results, the study presents some limitations. First, the analysis focused on a single case study, limiting the possibility of generalizing the findings. Moreover, the absence of digital tools prevented the automatic collection of data and the immediate validation of responses, making the analysis process more complex and less immediate. Another limitation concerns the short duration of the intervention, which does not allow for the observation of long-term effects on students' learning.

In light of these considerations, future research could further explore the comparison between digital tools and analog teaching strategies, in order to combine the advantages of both approaches.

In conclusion, the study contributes to current research on outdoor mathematics education and mathematical modelling by showing that meaningful math trails can also be implemented in contexts where digital technologies are unavailable or restricted. The findings highlight the importance of contextualized task design, cooperative interaction, and scaffolding strategies in supporting modelling processes within outdoor learning environments. More broadly, the study extends existing research on digital math trails by demonstrating how some of their core educational features can be maintained through carefully designed non-digital adaptations. In this sense, the findings contribute to current discussions on situated and embodied approaches to mathematical modelling in outdoor learning environments.

Conflict of interest

The authors declare that there are no conflicts of interest.

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