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Learning probability at grade 2: exploring the role of drawings for children's understanding

Chiara Andrà[1a], Domenico Brunetto[b], and Maria Polo[c]

[a]Dipartimento per lo Sviluppo Sostenibile e la Transizione Ecologica
Università degli Studi del Piemonte Orientale "Amedeo Avogadro", Vercelli, Italy
`chiara.andra@uniupo.it`
[b]Dipartimento di Matematica
Politecnico di Milano, Milano, Italy
`domenico.brunetto@polimi.it`
[c]Dipartimento di Matematica e informatica
Università degli Studi di Cagliari, Cagliari, Italy
`mpolo@unica.it`

Abstract: One of the challenges of Mathematics Education in primary school is to introduce probabilistic thinking when the understanding and use of ratios is not yet well developed. Seminal studies in Mathematics Education about probabilistic thinking focus their attention on misconceptions. Misconceptions refer to intuitive thinking and are generally understood as flaws in logical thinking. Notwithstanding the importance of this form of thinking, our study draws on an alternative approach to probabilistic thinking, which shifts the attention from the mere realm of intuitions to the intertwined nature of intuitions, procedures and representations. More specifically, we resort to storytelling and drawing to involve children in probabilistic activities. Within the MaTEs Italian project, a task on probability is assigned to grade-2 students, who are invited to make a drawing and to write their answers. The drawings have been coded according to the degree of narrative and mathematical elements they represent, and the written justifications provide insights on children's reasoning. The analysis of children's drawings reveal that also children at this young age are able to identify and employ the mathematical features of a probabilistic task in order to answer it correctly, but the majority fails to provide a written justification for their (correct) choices.

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[1] Corresponding author.

1. Introduction and literature review

Since the turn of the millennium, there has been a noticeable and accelerating global trend toward incorporating probability education into early school curricula, often before students have mastered the concept of ratios, which is traditionally considered fundamental to the classical and frequentist approach to probability (Borovcnik & Kapadia, 2014). This shift is primarily driven by the escalating importance of probabilistic reasoning across a multitude of fields, including economics, business, the natural and social sciences, and political decision-making. This curricular evolution has, in turn, arisen a vigorous academic debate and research agenda focused on the fundamental nature of probability and the most effective pedagogical strategies for its learning and teaching [5].

In response to these new challenges, numerous theoretical models have been developed to delineate the process of probabilistic thinking (for a comprehensive review, see [11]). These models typically analyze probabilistic reasoning by identifying, categorizing, and assessing the presence or absence of specific (partial) conceptions in students. A significant portion of this research has focused on explaining learners' difficulties by attributing them to the existence of deep-seated misconceptions or to missing or incomplete conceptual frameworks. It is crucial to note that the findings from these pioneering studies are often context-bound, focusing on discrete probabilistic concepts. For instance, Bar-Hillel and Wagenaar [3] meticulously investigated people's intuitive notions of randomness using both judgment tasks – such as evaluating the likelihood of a specific series of outcomes from a fair die – and production tasks, where participants were asked to generate a seemingly "random" sequence. This work revealed common intuitive biases, which Falk et al. [9] further characterized as a tendency toward "equal total frequencies" (the belief that all outcomes must appear equally often in a short run) and an "excess of alternation" (the tendency to switch between outcomes more frequently than is statistically likely, reflecting an over-compensation for perceived patterns).

Within this complex framework of probability learning, the prospect of engaging very young children with probability concepts, particularly in informal settings such as a summer break program, initially presents itself as a significant pedagogical obstacle – a seemingly "mission impossible." However, alternative theoretical approaches offer a path forward. Andrà and Stanja [2] advocate for the adoption of a cultural semiotic perspective on probabilistic thinking. This perspective fundamentally reframes probability not merely as a set of mathematical rules, but as a rich, historical, and dynamic "cultural product." This approach is deeply informed by the work of the philosopher and mathematician Brian Rotman, who proposes a foundational characterization of mathematics itself "as a practice, as an ongoing cultural endeavor" [13, p. 3]. By embracing this framework, Andrà and Stanja [2] emphasize the crucial mediating role played by artifacts and signs in the communication and construction of probabilistic understanding. Consequently, their research allows us to hypothesise that the deliberate use of visual representations, specifically children's own drawings, can serve as powerful cognitive tools that actively support children's problem-solving processes when they are confronted with probabilistic tasks.

This background establishes the context for the core inquiry of the present research, which seeks to contribute to the existing literature regarding modeling and drawings.

The research questions guiding this investigation are:

(i) Which kinds of drawings emerge when children are engaged in tasks that necessitate probabilistic thinking?

(ii) How do these various drawings function to support and mediate the meaning-making process for the children involved?

To address these research questions, our study employs a combined theoretical approach. It relies on a theoretical framework concerning the analysis and function of children's drawings, and simultaneously anchors its understanding of the mathematical content – probability – in Rotman's conception of mathematics as a cultural practice. These theoretical underpinnings, essential for the analysis and interpretation of the empirical data, will be further elaborated upon in the subsequent section of this paper.

2. Theoretical framework

Our study is substantially grounded in the conceptualizations of Rotman, whose work provides a crucial lens for examining the core subject matter. Our theoretical framework deliberately incorporates and builds upon Zan's understanding of narrative thinking. This reliance on Zan's perspective is particularly vital as it directly connects to the methodology of the study, specifically concerning the interpretation and utilization of drawings as a medium for expressing and analyzing thought processes. By integrating these two major theoretical streams, the research aims to establish a foundation for a nuanced and comprehensive analysis.

2.1. Rotman's frame: ideas, symbols and procedures

Rotman [13] posits that the endeavor of ' doing mathematics' is fundamentally underpinned by the coordination of three distinct, yet interwoven, theoretical discourses: Idea, Symbol, and Procedure [13, p. 1676]. Mathematical thought, according to this perspective, necessitates the fluent coordination among these three components. In fact, Idea is the domain of intuition and human thought; Symbol is the domain of signs, communication and objectification; and Procedure is the domain of action, transformation and operation.

More specifically, the domain of Idea is synonymous with human thought, rooted in the individual's conceptualization and articulation, often initially expressed through natural language narratives [13]. It constitutes the core intuitive understanding of mathematical concepts. Within the specific context of probability, Andrà and Stanja [2] interpret Idea as the realm of intuitive approaches to uncertain situations and fundamental concepts such as variability. This domain is where an individual's pre-mathematical, often spontaneous, understanding takes shape. Furthermore, Andrà and Santi (2011) characterize the intuition of mathematical concepts as an experience where access to the object's distinctive features is self-evident, coercive, and global. This immediacy and self-evidence are linked by Andrà and Santi (referencing Radford, 2008) spatial-temporal, sensorimotor and perceptive activity that semiotic means of objectification accomplish, support, foster" (Andrà & santi, 2011, p. 115). Consequently, the domain of Idea concerning probability is inherently situated within space and time, serving as the experiential ground from which intuitions emerge and are subsequently accounted for through narrative expression.

The journey from personal intuition to shared mathematical practice – the development of personal and collective knowledge – is intrinsically linked to the productive engagement with artifacts as instruments for knowledge-building. This transition leads directly to the domain of Symbol. Rotman [13] defines this as the domain of signs, encompassing all forms of communication and semiotic practices, ranging from specific notational devices (like variables or operators) to entire linguistic systems. The profound significance of symbols in the process of mathematics learning is underscored by Duval [8], who asserts that *noesis* without *semiosis* is impossible; that is, any process of meaning-making inherently requires a system of signs to materialize. In the realm of probability, Andrà and Stanja [2] define the Symbol domain as the comprehensive set of signs and symbols conventionally employed, including Venn diagrams, algebraic formulas, tables, histograms, and other visual or symbolic representations. Crucially, a sign is understood as an artifact that has been culturally imbued with a meaning, serving as a representation for a mathematical idea. For an artifact to transcend its mere physical existence and function as a mathematical sign [14], it must establish a meaningful relationship with a corresponding mathematical Idea. In elementary probability, this often involves the critical translation from the physical domain of artifacts (situated in space and time, such as dice or spinners) to the abstract domain of numbers and symbols. The use of a simple artifact like a spinner can evoke two pivotal cultural conceptualizations of probability: from one side, the classical/Laplacian probability (i.e., the notion of probability as a ratio between favorable outcomes and the total number of possible outcomes); from the other side, the frequentist probability, namely the understanding that probability is assigned based on the relative frequency of an event over a series of repetitions, an understanding particularly salient with common, repeatable artifacts (e.g., dice, cards, coins, spinners), as highlighted by the condition that "probability can be assigned only to an event that can be repeated" [10, p. 2].

Procedure encompasses the domain of purposeful actions, processes, and operations performed on and with artifacts and signs [13]. This resonates strongly with the perspective of Duval [8], who identifies mathematical learning as the progressive capacity to execute correct, meaningful actions on mathematical signs. Mathematical actions are fundamentally transformations, which can manifest in two key ways: (i) treatment – a transformation that occurs *within* the same semiotic register, for example algebraically manipulating one formula into an equivalent but distinct form; (ii) conversion – a transformation that occurs *between* two different semiotic registers, for instance converting an algebraic formula into its graphical representation, or vice versa. While Procedures can, and ideally should, be mathematically explained or justified by the underlying Ideas, Rotman notes that they may sometimes be performed purely algorithmically, without explicit reference to or reflection upon those foundational Ideas.

2.2. Drawings and the power of narrative in Mathematics Education

The integration of storytelling methodology in the mathematics classroom, often referred to as storytelling in mathematics, provides a powerful means for students to construct meaning and develop a deeper connection with the subject matter. Stories, fundamentally, serve as a robust framework for organizing, transmitting, and interpreting information, allowing individuals to create personal and contextual meaning in their lives. A critical feature that distinguishes stories from other forms of narration, such as historical accounts or scientific reports, is their intrinsic capacity to direct and engage our feelings toward the content being shared. In essence, stories evoke an emotional response. This emotional engagement, coupled with the stimulation of students' imagination concerning the curriculum content, is precisely where the significant educational value of stories lies [17]. In an attempt to connect these theoretical considerations with Rotman's [13] model, we can say that storytelling in mathematics is related to the realm of Idea, i.e. the intuitive grounding of mathematical concepts.

The pedagogical application of storytelling in mathematics utilizes different types of narratives, each designed to convey a mathematical concept in a specific manner. Among them, the so-called question-posing stories are narratives that are structured to generate a mathematical problem or inquiry. In primary education, this category is often approximated by "word problems" found in textbooks, which are intended to emulate a story-based questioning [17] efficacy of the "question-posing story," particularly as manifested in the traditional word problem, has often been severely diminished. All too frequently, these "problems" are stripped of the rich, well-organized narrative details that are essential for engaging and motivating students and for shaping the emotional response of the reader or listener. Consequently, they lose their power to foster genuine engagement. While there is widespread agreement that contextualizing mathematical problems within concrete, familiar, and realistic situations benefits children both motivationally and cognitively, the prevalent use of concise, artificial, and often falsely relatable texts proves counterproductive. Rather than fostering comprehension and engagement, these poor-quality texts inadvertently encourage the use of "cognitive shortcuts." Students learn to employ a selective reading strategy, focusing only on the search for numerical data and mathematical "key words" [16]. This pattern of interaction reinforces the damaging misconception that mathematics is merely a sterile collection of procedures to be applied without true meaning or conceptual understanding. In Rotman's [13] words, we could say that this kind of approach fosters the operational capability of Procedure, yet risks disconnecting it from both Idea and Sign.

To counteract these negative effects, the narrative context must be rich and profoundly meaningful for the students, namely, in Rotman's [13] words, it emerges a need to connect Idea and Procedure. As suggested by Zan [15], the mathematical questions must arise "in" the story itself, organically growing out of the plot and the protagonists' motivations, rather than being artificially imposed "about the story." The problem must be of genuine interest to the characters within the tale. This requirement highlights a significant contrast with traditional classroom problems, which are often "externalized" – meaning the person who poses the problem is different from the person who is expected to solve it, a dynamic unlike real-life problem-solving scenarios.

Furthermore, comprehending a narrative engages what Bruner [7] narrative thinking, which he contrasted with paradigmatic or logical-scientific thinking. Narrative Thinking is focused on interpreting human actions, people, their intentions, and feelings, and its output is a plausible and reasonable story. Logical-Scientific Thinking is dedicated to categorizing reality, recognizing the order of things, and its output is demonstrative arguments and logical proofs.

While Bruner argued that these two modes of thought are fundamentally irreducible, the distinction should not be interpreted as narrative thinking being an impediment to learning or teaching mathematics. On the contrary, the strong presence of narrative thinking in everyday life and its complementarity with logical thinking means that it can actively support the development of logical thought processes [16].

This synergistic relationship is particularly effective in what Rosetta Zan refers to as "story problems" – those mathematical problems that successfully incorporate the rich, meaningful, and protagonist-driven characteristics described above. In these contexts, the two types of thinking – narrative and logical-scientific – can effectively support each other, thereby deepening students' comprehension of abstract mathematical concepts. The story provides the human, contextual, and emotional foundation, while the mathematics provides the logical framework for resolution.

A crucial distinction should be made between the different types of information embedded within a word problem. A problem contains information necessary for the construction of a mental representation (the 'story' data) and information necessary for the execution of the mathematical solution (the 'solution' data). As emphasized by Zan [15], "the point is that the data a child needs to represent the problem are not necessarily the data he or she needs to use in the solution" (p. 6). This highlights a potential cognitive gap: students must first successfully construct the narrative model, and then transition to identifying and utilizing the quantitative data relevant for the mathematical operation. A successful problem-solving process requires two critical alignments: (i) alignment of solution-relevant information and (ii) alignment of representation-relevant information. The former can be understood as the quantitative, operative data that should be clearly and logically connected to the established narrative structure. If the data points feel arbitrary or divorced from the story, the student's ability to apply them correctly is diminished. With respect to the latter, the elements used to build the narrative representation must be logically consistent and coherent, allowing the student to form a stable and understandable mental model of the situation.

The integration of drawings when the students deal with word problems is a powerful yet underexplored area of research, particularly concerning how different visual representations support distinct cognitive and communicative skills [1]. These drawings serve a crucial mediating role, bridging the gap between stringent mathematical rigor and the narrative sensemaking essential for complex problem-solving. Drawings can offer access to Rotman's [13] domain of

Sign and their representational power in mathematics. Visual tools act as a vital link between real-world contexts and abstract mathematical structures, connecting Idea with Procedure [13].

Despite a relatively limited body of research, existing studies have demonstrated a positive correlation between the strategic use of drawings and enhanced mathematical modeling performance [12, 4]. Specifically, the work of Rellensmann et al. [12] highlights that students' strategic knowledge of drawing is positively associated with their modeling performance. This relationship is further nuanced and mediated by the *type* and *accuracy* of the drawings generated. Their findings suggest a complex relationship: while the accuracy of *situational drawings* is only indirectly related to performance, there is a clear and robust correlation between the accuracy of *mathematical drawings* and overall modeling success. Conversely, Bassi and Brunetto [4] suggest that this strict demarcation between drawing types and their impact may become less distinct when students are engaged in more sophisticated and open-ended modeling activities, implying that the boundaries between situational and mathematical drawing functions can be creatively overcome in complex tasks.

Building upon existing literature [12] and the insights derived from the findings of the research project of which this study is part, drawings have been identified, each characterized by a unique function and level of abstraction. The first one is situational drawing. This type of drawing visually depicts the objects and context described in the problem exactly as they would appear in a real-world setting. It is an immediate, direct representation of the problem's narrative environment. It involves a low degree of abstraction, as the primary goal is fidelity to the physical reality of the problem elements. In Rotman's [13] terms, it is as if the student is accessing the realm of Idea through the narrative of the problem, its characters and its elements.

The second category is mathematical drawing. It is highly selective, including only the essential elements of the problem that are directly relevant to constructing the mathematical solution. The objects within the drawing are simplified and idealized to represent their core mathematical properties (e.g., a car might become a point mass, a building a simple rectangle). This type is significantly more abstract. It fundamentally functions as an externalization of the mathematical model itself, translating the real-world scenario into a structured, solvable mathematical framework. In Rotman's [13] terms, we could say that the realm of Idea becomes, in this case, a blending of both narrative and mathematical elements and this can offer access to Procedure.

The third category is narrative drawing, which is defined as a specific sub-type of situational drawing. The narrative drawing incorporates elements that aid the student in constructing meaning from the narrative problem context. While it remains rooted at the situational level, its choice of visual emphasis and inclusion suggests an active engagement with the problem's underlying logic. As conceptualized by Zan [15], the designation "narrative" underscores the fact that these drawings, while situational, demonstrate the activation of logical thinking. This logical processing is often supported and facilitated by narrative thinking, which helps guide the meaning construction process and the initial comprehension of the problem's demands.

The fourth category is dynamic mathematical drawing, which is configured as a specific sub-type of mathematical drawing, the dynamic mathematical drawing includes conventional or intuitive graphical signs (e.g., arrows, vectors, trajectories) that are capable of highlighting a movement, change, or state-transition of the represented objects. This movement or evolution within the drawing is directly associated with the progression or change over time inherent in the problem-solving process itself, making the drawing a visualization not just of the model, but of the *process* of reaching a solution. In other words, dynamic mathematical drawings allow for a connection between Procedure and Idea, reducing the risk of pure algorithmic performance.

3. Methodology

The study involved a cohort of 363 Italian pupils who had recently completed Grade 2 of primary school. Data collection was carried out during the 2024 summer break, the period spanning between the completion of Grade 2 and the commencement of Grade 3. This activity was integrated into the required summer homework assigned by their respective teachers, fostering participation and relevance to the curriculum.

A stratified sampling method was employed across various Italian schools as part of an Italian National project funded by the Ministry of Education. Following the sampling, school deans were formally requested to invite their Grade 2 teachers to participate in the project. 95% of the invited teachers consented to participate.

Given that data collection occurred during the summer when schools were closed and direct teacher-pupil interaction was suspended, a web app was developed specifically for the project. This application served the crucial dual purpose of presenting the mathematical tasks to the children and facilitating the collection of their work. Critically, the children's elaborations were made using traditional paper and pencil, which were then uploaded via the app, bridging the gap between physical execution and digital collection.

Consensus for collecting anonymous children data has been obtained from their parents via a form, distributed by the schools.

The core of the study involved six distinct mathematical modelling activities. Each activity was designed to commence with an engaging story, providing a context for the subsequent tasks. As the activity progressed and different tasks were assigned, new fragments of the story were revealed, maintaining engagement and building a narrative structure.

The very first task across all activities consistently required the pupils to "draw the story you heard." This initial task aimed to capture the children's interpretation and visual representation of the narrative context before engaging with the mathematical problem-solving.

This particular paper zeroes in on the first two tasks of the first activity. Thematic focus for the first activity is the topic of candies, suggesting an accessible and relatable context for the young participants. Activity 1, like the others, starts with an introductory story to set the scene for the subsequent tasks:

> Every time she visits her grandchildren, Elisa and Matteo, Grandma Adele brings candies, which the children are supposed to put in the kitchen drawer and then to eat candy every now and then.

The story goes on, setting the first task:

> Today, Grandma arrived with three bags – one green, one white, and one red – and she said to the children: "The green one contains only cherry candies. The white one contains four mint candies and three orange candies, while the red one contains three mint candies and four orange candies."

Then, the first task consists in drawing the story already heard.

The second task is as follows:

> Matteo likes orange candies most. Which bag is best for him to choose from? Why?

We undertook an analysis of the children's drawings produced during the execution of the first two tasks. This analysis is framed by and interpreted through the theoretical lenses described in the preceding sections of this paper. In particular, we firstly classify the drawings made in the first task into one of the four aforementioned categories. We, thus, focused on the elements that the children report in their drawings: if a character and/or an object that is a strong narrative element but is not, or partly, related to the mathematical solution of the problem, is drawn, we started with labelling the drawing as situational or narrative, and then we look for mathematical elements. If none is present, the drawing is labeled as situational, otherwise it is labeled as narrative. If mathematical elements are present and narrative ones are absent, the drawing is labeled as mathematical. After this first classification, a detailed analysis of the content of the drawing is carried out. This classification helps us find an answer to the first research question about the kinds of drawings that emerge in a task involving probabilistic thinking. Then, we look at how the students deal with the second task and we attempt an answer to the second research question about how the different drawings can support the meaning making process.

4. Results

This section details the findings from the analysis of student work across the two aforementioned tasks designed to examine their understanding and representation of probabilistic scenarios. We first examine the drawings produced during Task 1, classifying them into four categories: situational, narrative, mathematical, and dynamical mathematical. Subsequently, we investigate how the same students approached Task 2, which required them to select the most advantageous bag for Matteo, a boy who likes orange-flavored candies.

The initial analysis focuses on the drawings created by the students to interpret the problem description. A significant portion of the drawings fell into the situational and narrative categories, indicating an early focus on the contextual and story elements of the problem rather than the underlying mathematical structure. Figure 1 provides compelling examples of situational drawings. The drawing reported in Figure 1 (left) reproduces the story's setting, depicting "grandma Adele” holding three distinct bags: one white, one red, and one green. The drawing is a faithful visual report of the textual narrative, devoid of any attempt to represent the quantitative details (the number of candies) or the probabilistic nature of the situation. Crucially, the student's approach to Task 2 confirms this lack of mathematical engagement; they submitted the *exact same drawing* without any modification or accompanying mathematical reasoning. This response is interpreted as a failure to solve the task, as it demonstrates an inability to translate the situational context into a mathematical decision-making framework.

Figure 1. Two examples of situational drawings.

The drawing reported in Figure 1 (right) is categorized as situational, as it includes the core objects – the three bags (green, white, and red) – but attempts a rudimentary representation of the contents. The student has drawn candies of various colors (pink, green, and orange) scattered around the bags, intending to suggest the distribution of flavors. Specifically, the visual arrangement suggests an approximation: two pink candies with the green bag, three green and three orange candies with the white bag, and another three green and three orange candies with the red bag. It is noteworthy that the *number* of candies drawn does not precisely match the numbers specified in the original story prompt. This suggests that the student is prioritizing the *qualitative* representation of the situation (i.e., *what* flavors are in *which* bag) over the *quantitative* accuracy necessary for a probability calculation.

The transition from drawing the scenario (Task 1) to solving the probability question (Task 2) reveals different levels of cognitive shift. For students whose drawings were purely narrative, the transition often failed, as seen in the Figure 1 (left) example. The student who made the drawing reported in Figure 1 (right) offered the following written justification for their choice in Task 2:

> The orange is orange, hence the candies are in the orange bag.

This response is highly problematic and indicative of a profound conceptual misunderstanding. The student appears to be making a non-mathematical, purely linguistic association ("orange-flavored" implies "orange bag"). This interpretation is flawed on two key counts: on one side, it confuses the color of the *flavor* (orange-flavored candies) with the color of the *container* (the bag), and on the other side the premise is incorrect, as the story explicitly states there is no orange bag among the three options (only white, red, and green).

Therefore, we conclude that this response, rooted in a color association rather than a probabilistic comparison of the bag contents, is incorrect and highlights a critical barrier in moving from a concrete visual representation to abstract mathematical reasoning.

To recall, a narrative drawing incorporates elements from the problem's context or story, thereby aiding the student in constructing meaning. Crucially, while this type of drawing remains anchored at the *situational level* – depicting the objects and characters of the story – its choice of visual emphasis and the inclusion of specific, non-mathematical details signals an active, albeit often implicit, engagement with the problem's underlying mathematical nature. Figure 2 showcases the work of a single student as they tackled Tasks 1 and 2. The drawing corresponding to Task 1 (Figure 2 left) is highly descriptive and meticulous in its representation of the problem. It depicts three bags: in the green bag, the student has included a symbol of a cherry. Notably, closer inspection reveals evidence of revision: the student had initially drawn two pink candies inside this bag but subsequently erased them. This erasure suggests a process of cognitive checking and correction, pointing towards a generalisation. The white bag contains four green (assumed mint-flavored) and three orange (assumed orange-flavored). Similarly, the red bag contains seven candies, but the ratio is reversed: three green candies and four orange candies.

A significant feature distinguishing this narrative drawing from purely *situational* drawings (like the one discussed in Figure 1, right) is the containment and accurate enumeration of the objects. The candies are explicitly drawn *inside* the bags and are counted correctly according to the problem statement. This careful and accurate representation of the quantities and their locations demonstrates a strong effort to map the text onto a concrete visual model, a necessary precursor to formal mathematical calculation. The drawing for Task 2 (Figure 2 right) shifts focus from the static containers to an active, human element. It features a recognizable character – Matteo – portrayed with a seemingly happy or determined expression. The key action depicted is Matteo taking the red bag. Our interpretation of this visual choice suggests a sophisticated, if unstated, inference on the student's part. In the context of a probability problem that likely asked about the chance of getting an orange-flavoured candy, the red bag is the container that offers the highest probability of success, as it holds the greatest number of orange candies (four orange versus three mint). Therefore, by drawing Matteo choosing the red bag, it seems that the student is visually encoding a mathematically-informed decision.

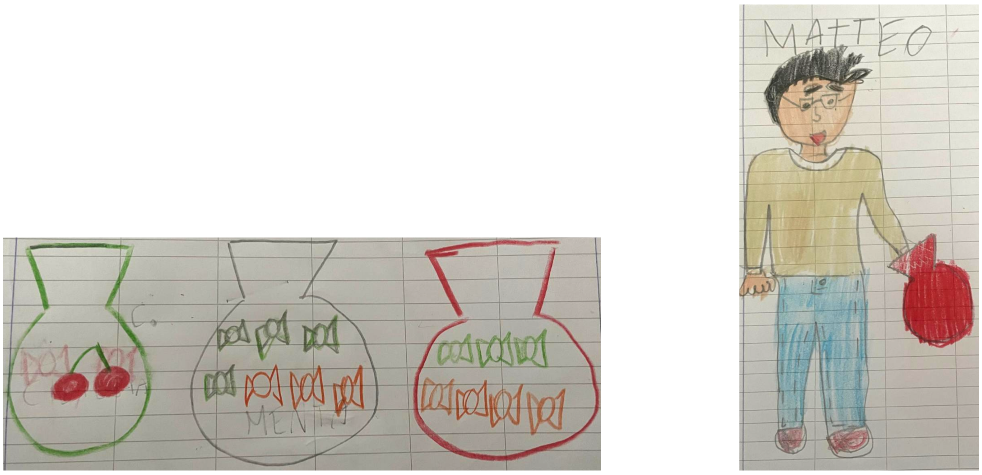

Figure 2. An example of a narrative drawing. To the left, the answer to Task 1. To the right, the answer to Task 2.

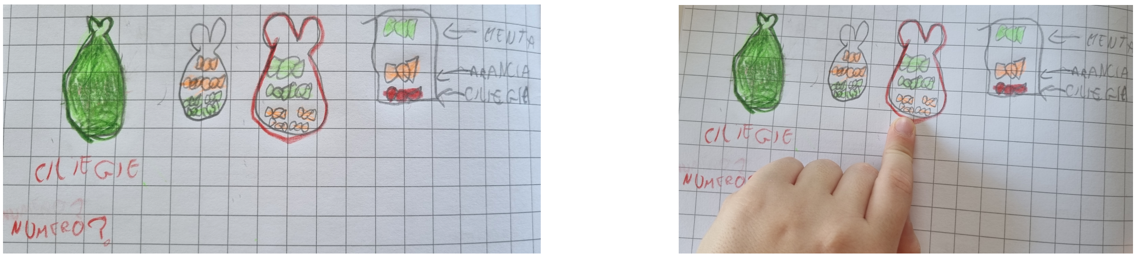

Figure 3. Another example of a narrative drawing. To the left, the answer to Task 1. To the right, the answer to Task 2.

Another instance of a narrative drawing, rich in situational context yet containing crucial elements for the mathematical problem's resolution, is presented in Figure 3. This figure illustrates the same student tackling both Task 1 and Task 2. Focusing firstly on the representation for Task 1, shown in Figure 3 (left), the student employs distinct visual markers to represent the components of the problem. They draw a prominent green bag. Attached to this green bag is a handwritten question: “Ciliegie, numero?” which translates to “Cherries, number?". This annotation is significant because it highlights the student's awareness that the initial problem statement is incomplete or ambiguous regarding the quantity of cherry-flavoured candies. Adjacent to this clearly green bag, the student has drawn two other bags: one in white and one in red. The contents depicted within these bags bear a strong resemblance to the visual representation provided by the same student in Figure 2, suggesting a consistent mental model for the problem's setup. To clarify the connection between the colours and the candy flavours, the student has included a legend or key: green is assigned to ’ mint’ flavour, orange is associated with ’ orange’ flavour, red is designated for ’ cherry’ flavour. This visual and textual organization confirms the student's method of identifying and compartmentalizing the different variables involved in the problem – the colours (representing flavours) and the associated quantities.

Moving to the student's engagement with Task 2, depicted in Figure 3 (right), we observe the student's own finger pointing directly towards the red bag. Given the structure of the problem (as previously discussed in the context of Figure 2), the red bag is, in fact, the correct item to identify for the solution to Task 2. However, while the *selection* is implicitly correct based on the underlying mathematical nature of the task, the drawing itself and the moment captured lack an explicit, articulated justification for this choice. This common pattern, where a student correctly identifies the answer visually but fails to provide the reasoning, suggests that the drawing serves as an intuitive or situational model rather than a fully developed mathematical proof. The visual element confirms the choice but leaves the mathematical justification unstated, mirroring the observational findings from the analysis of Figure 2. In Rotman's (2003) terms, we could say that the Idea is fully developed, but the Signs lack for language tools to express the correct intuition.

We conclude our analysis of students' drawings by examining three exemplary solutions to Task 1, presented in Figure 4, which distinctly prioritize mathematical and numerical elements over narrative ones. These examples showcase a significant reduction in storytelling to focus on the problem's mathematical elements. The student's drawing on the left of Figure 4 features three distinctly colored bags – green, white, and red – each clearly labeled with its contents. The narrative is limited but crucial for identifying the *type* of candy. A speech bubble indicates the content of the green bag as "caramelle ciliegia" (cherry candy). The presence of only cherry candies in this bag is clear for the student. The white bag is labeled with the quantities and types of candies: "caramelle: 3 arancia, 4 menta" (candies: 3 orange, 4 mint). Similarly, the red bag is labeled "caramelle: 4 arancia, 3 menta" (candies: 4 orange, 3 mint). We can see the emergence, in this drawing, of a transition from a purely visual narrative to a coded representation, where written text acts as the primary carrier of essential numerical information.

The drawing in the middle of Figure 4 maintains the visual presence of the three bags, arranged on what appears to be a table. The numerical and descriptive information is separated from the bags and linked to them using arrows, suggesting a more formalized symbolic mapping. Specifically, the descriptor "solo ciliegia" ("only cherry") is connected to the green bag with an arrow, reinforcing the qualitative nature of its contents. The writing "4 menta + 3 arancia" (4 mint + 3 orange) is connected to the white bag and we can see the emergence of a formal mathematical sign, namely the '+' sign, suggesting a summation or a collection of items, further emphasizing the numerical composition. The text "3 menta + 4 arancia" (3 mint + 4 orange) is connected to the red bag. In this drawing, the embedded narrative is reduced in favor of external, structured annotation, moving closer to a diagrammatic representation of the problem's data.

In the drawing to the right of Figure 4, the contents of the bags are depicted directly using color-coded dots, standing for the candies. An arbitrary number of red dots (representing cherry candies) are drawn inside the green bag. The lack of an explicit count underscores that the composition, rather than the total number, is the key mathematical feature (i.e., zero orange candies). The numerical composition of the white bag is visually concrete: 3 orange dots and 4 green dots are drawn inside it. Similarly, 4 orange dots and 3 green dots are drawn inside the red bag.

The differences in representational style across Task 1 does not correspond to a deeper conceptual understanding, as revealed by the students' answers to the subsequent Task 2 (Figure 5). The students who produced the drawings on the left and in the middle of Figure 4, despite their clear representation of the numerical data, provided solutions to Task 2 that were similar to those observed in Figures 2 and 3. This indicates they selected the red bag (the correct solution) but failed to provide a robust, mathematically sound justification for their choice. We can further notice that, for the drawing presented in Figure 5 (right), a significant linguistic and interpretative shift becomes evident. The accompanying written statement, "Ho scelto questo" ("I chose this"), is crucial, as it employs the first singular person ("I"). This use of the first-person pronoun marks a clear departure from the narrative perspective observed in Figure 2. In those instances, the characters and the actions described were framed in the third person, referring to Matteo. The student's statement "Ho scelto questo" signifies a fundamental change in the relationship between the student and the action being documented. A first-person interpretation and ownership of the choice or action performed emerges. In our view, this implies that the student is not simply documenting a scenario but is actively asserting their participation and decision-making process within the task, potentially offering insight into their metacognitive awareness or their personal connection to the mathematical or thematic content of the drawing.

The student who created the highly minimalist drawing on the right offered a tentative justification, writing "4 caramelle" ("4 candies"). While concise, this suggests an attempt to articulate the reason *why* the red bag was the optimal choice – namely, because it contained the maximum number of orange candies (4) compared to the other non-cherry bag (3). This answer hints at a developing concept of maximizing favorable outcomes, the core principle behind maximizing the chance of getting an orange candy. This correlation suggests that focusing on the purely numerical structure in the visual representation might facilitate a more analytical approach to the subsequent justification phase.

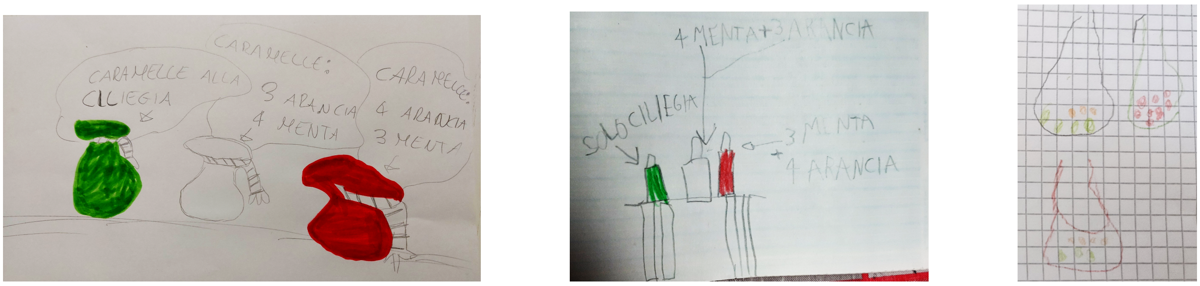

Figure 4. Three examples of mathematical drawings while students are dealing with Task 1.

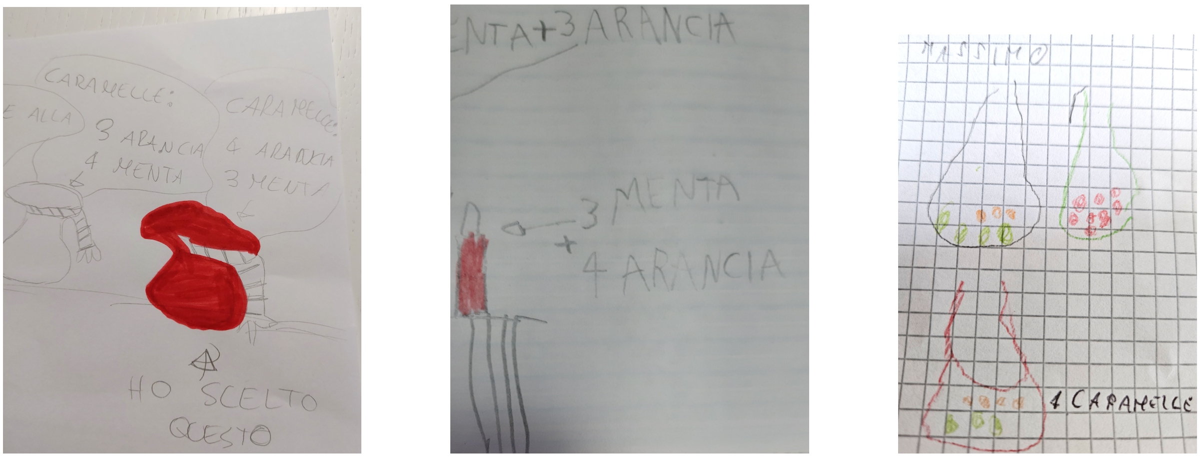

Figure 5. The same three students' mathematical drawings while dealing with Task 2.

5. Discussion

In this study, we showcased examples of situational, narrative and mathematical drawings taken from a large sample of 363 children who dealt with a probabilistic task about candies. We recall that the participants in our study are grade-2 students who have not been exposed to probability content at school before this activity, which is carried out during the summer break. Some drawings belong to the narrative category, as they are strongly related to the elements of the story (grandma Adele, the boy Matteo, the presence of candies of various colors), but they also represent the candies inside each bag, mirroring the quantities mentioned in the story and hence accessing the mathematical symbols and procedures necessary to solve the task mathematically. The students, in fact, correctly identify the red bag as the one that maximises the probability of getting an orange-flavoured candy, but their seemingly weak ability to represent, in their own words, their reasoning limits their ability to justify their (yet correct) choice. We employ Rotman's [13] framework to interpret this phenomenon: the intuitive realm of Idea is well supported by the story problem and the children's own drawings, but in the realm of Sign the children seem to lack the language tools to account for their choices. We also acknowledge the potential of distinguishing between situational and narrative drawing. The analysis of the examples in Figure 1 underscores the challenge students face in disentangling the narrative elements from the mathematical structure of probability problems. The tendency to remain within a situational framework often prevents them from identifying and comparing the relevant ratios of desired outcomes to total outcomes.

The observations about the drawings in Figure 2 highlights a fundamental characteristic of narrative drawings: they are not mathematical drawings. They do not use formal mathematical symbols, diagrams, or structures to convey the solution. Instead, the student embeds their reasoning and potential solution within the narrative itself. The visual story – Matteo's choice – becomes a proxy for the mathematical conclusion (the highest probability is associated with the red bag). The student does not feel the need to provide a written explanation or a numerical calculation because the answer is encoded within the chosen action of the drawn character. This feature underscores the distinction of narrative drawings as visual tools that illuminate a student's intuitive understanding and qualitative reasoning before or alongside the development of formal mathematical skills. We could say that the students stay in the realm of Idea [13].

In the mathematical drawings, a significant departure from the narrative elements emerges and more abstract representations are present. A number of the students who made this kind of drawing was able to (partly) justify their answer, as if the lightening of the cognitive load given by not feeling obliged to represent the narrative elements of the story too faithfully allowed them to concentrate on the mathematical elements, choosing symbols that allow them a more direct access to the mathematical ideas and therefore also to the words necessary to justify their choices. For these drawings, an entanglement of idea, Sign and Procedure [13] emerges.

The study adopts a descriptive and qualitative approach, seeking to identify the processes through which young children construct an understanding of probability by employing drawings and narrative thought. This methodology moves beyond the conventional focus on merely highlighting misconceptions. The analysis of drawings presents inherent complexity, as multiple interpretations by different researchers are plausible, and a single drawing may lend itself to various classifications. This potential ambiguity can be mitigated, firstly, by involving multiple researchers, and, more significantly, by redirecting the focus from simple classification towards an examination of how drawings function as pivotal components in the meaning-making process. Consequently, the primary objective of this research is not the creation of classifications, but rather the demonstration of the potential inherent in a methodology that emphasizes representations beyond verbal expression.

6. Conclusions

The essence of this research lies in employing descriptive and interpretive methodologies for drawing analysis, rather than mere correction, premised on the notion that children frequently lack the requisite semiotic tools to substantiate their choices. The pedagogical implications for mathematics teachers underscore the necessity of encouraging students to create drawings when addressing mathematical problems, as opposed to concentrating solely on verbal expression, the written text, and the procedural steps. Indeed, Rotman's theoretical framework posits that the integration of all these components collectively constitutes mathematical thinking in its entirety.

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